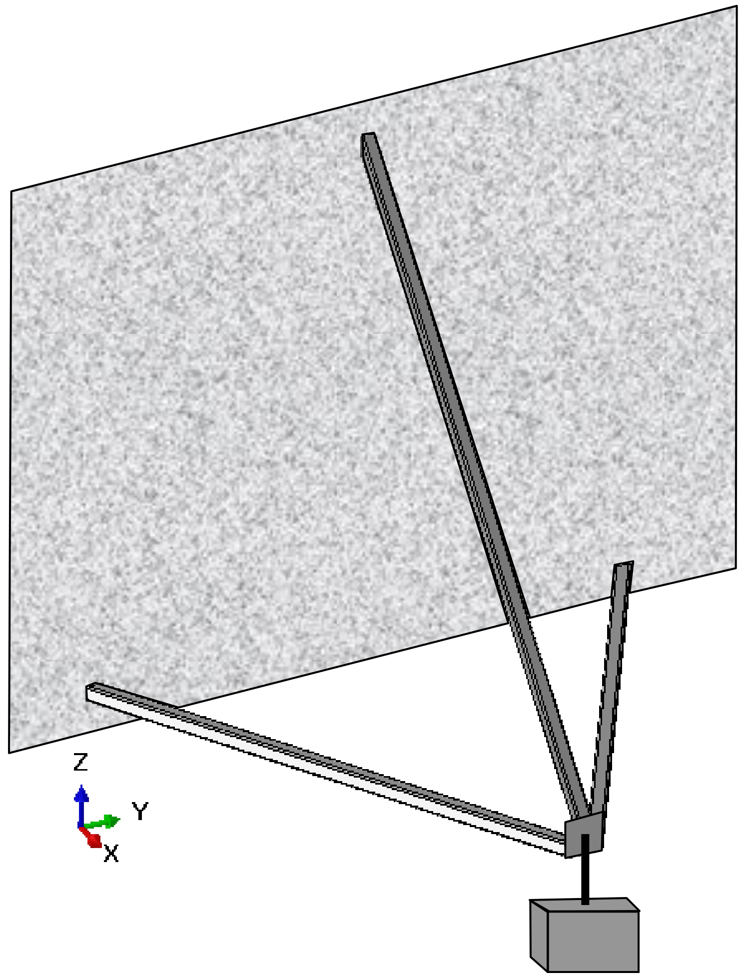
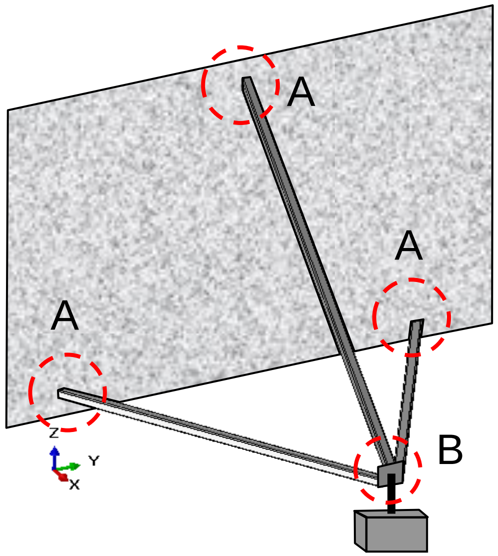
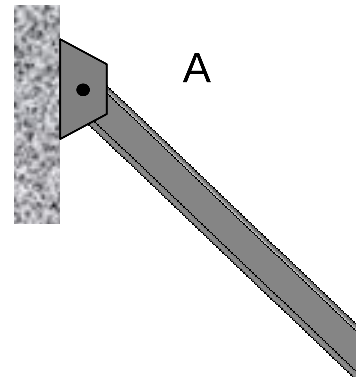
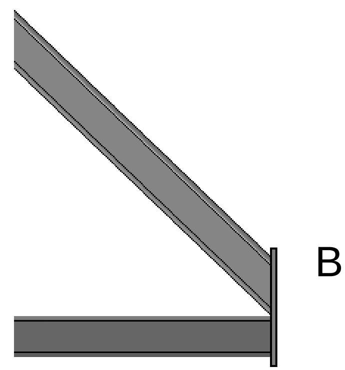
- primer reševanja volumskega mehanskega problema z MKE



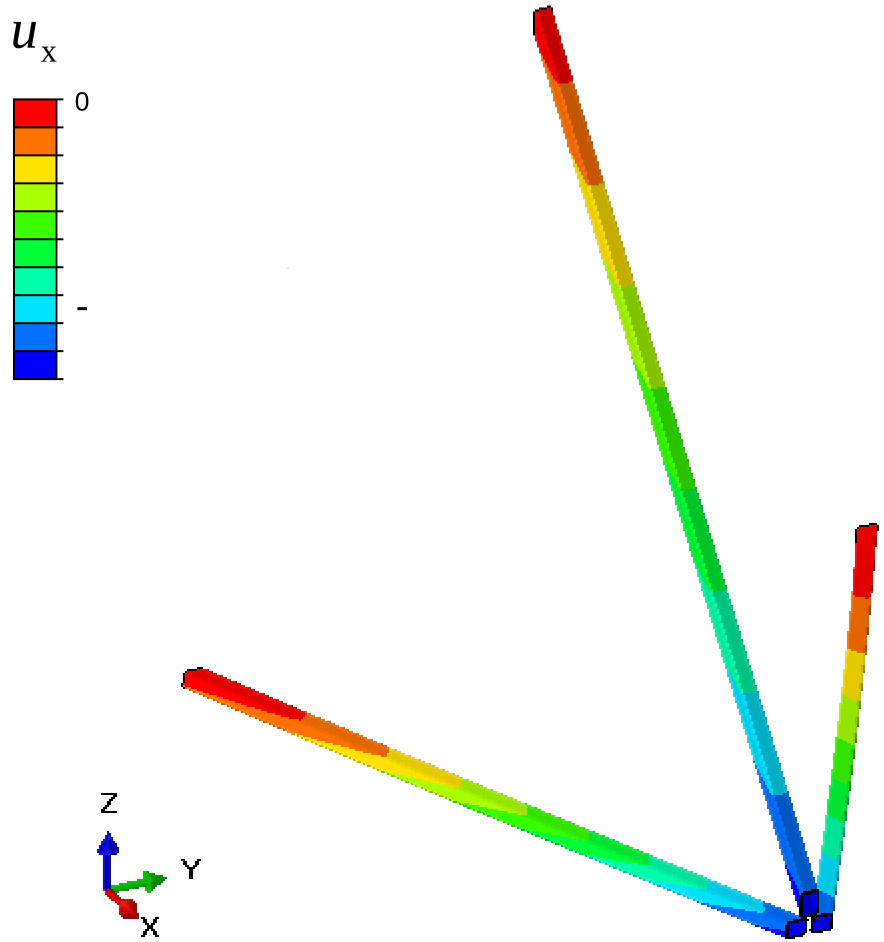
- izvedba pritrditve konstrukcije in poveze med konstrukcijskimi elementi



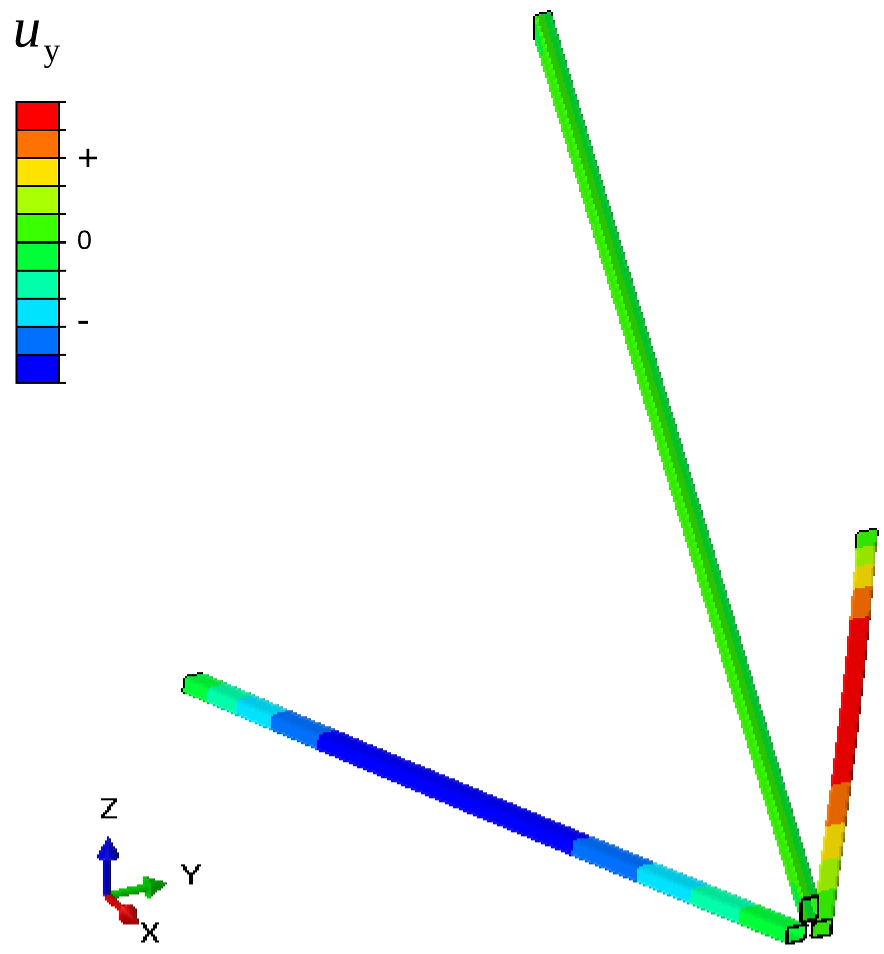




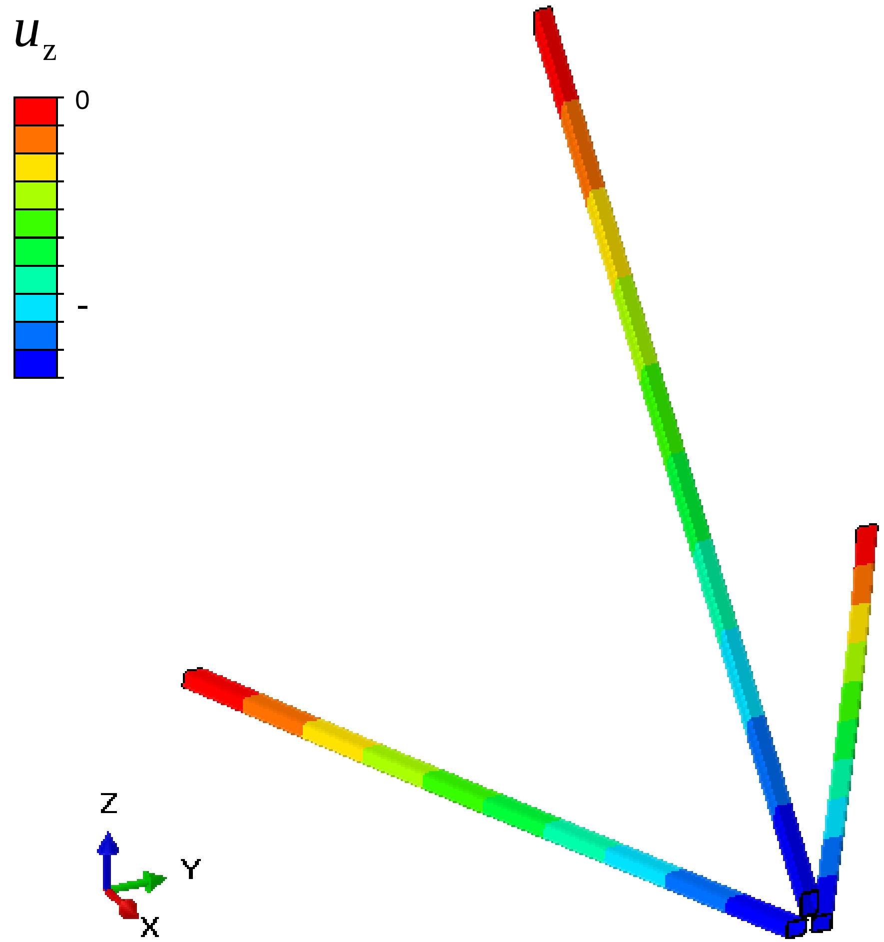
• pomiki v Kartezijevem koordinatnem sistemu



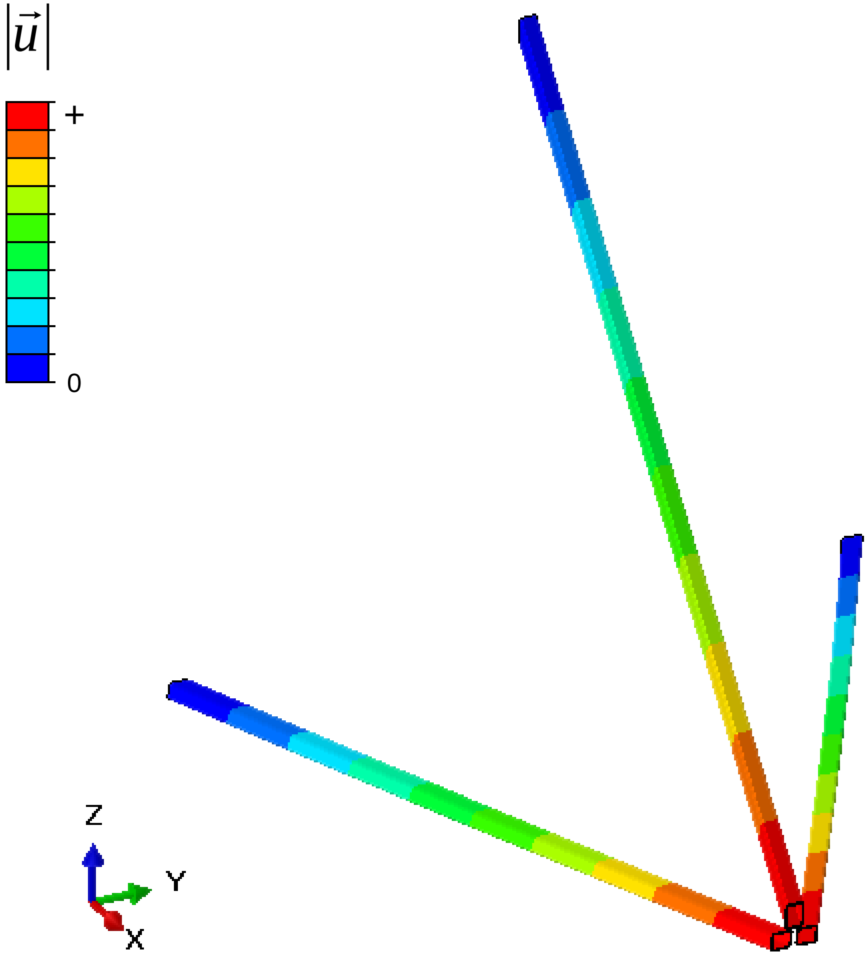
• pomiki v <u>Kartezijevem koordinatnem sistemu</u>



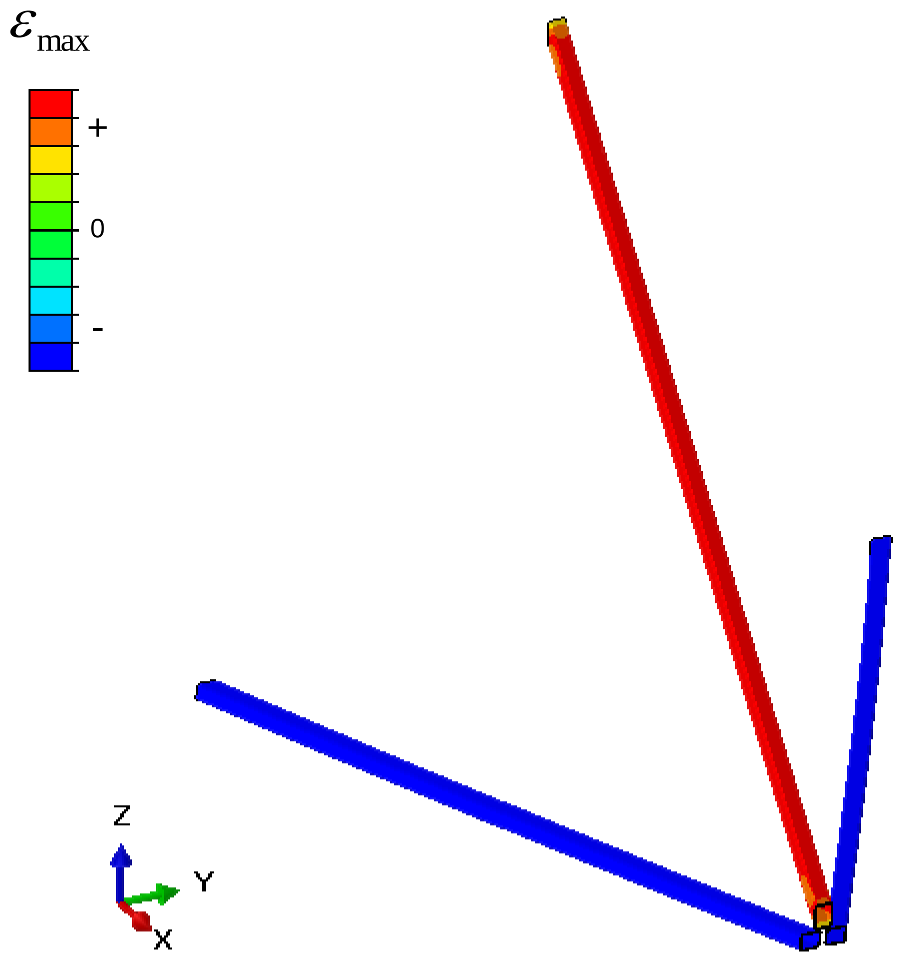
- pomiki v <u>Kartezijevem koordinatnem sistemu</u>



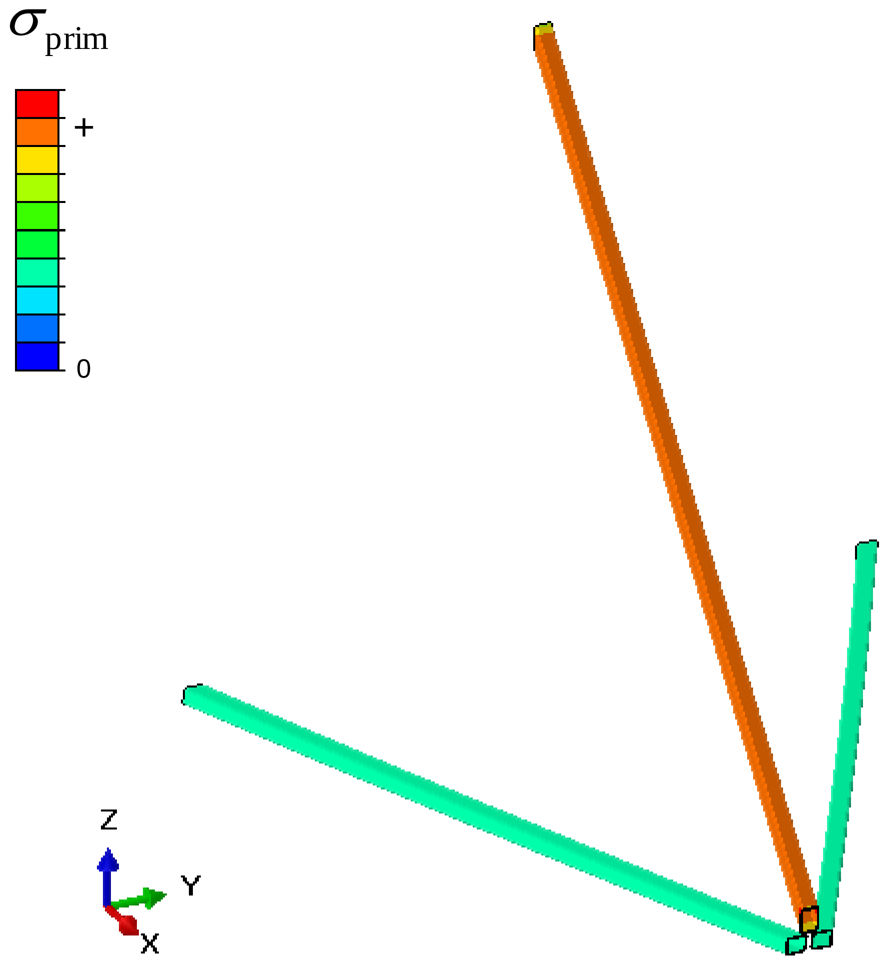
- pomiki v Kartezijevem koordinatnem sistemu



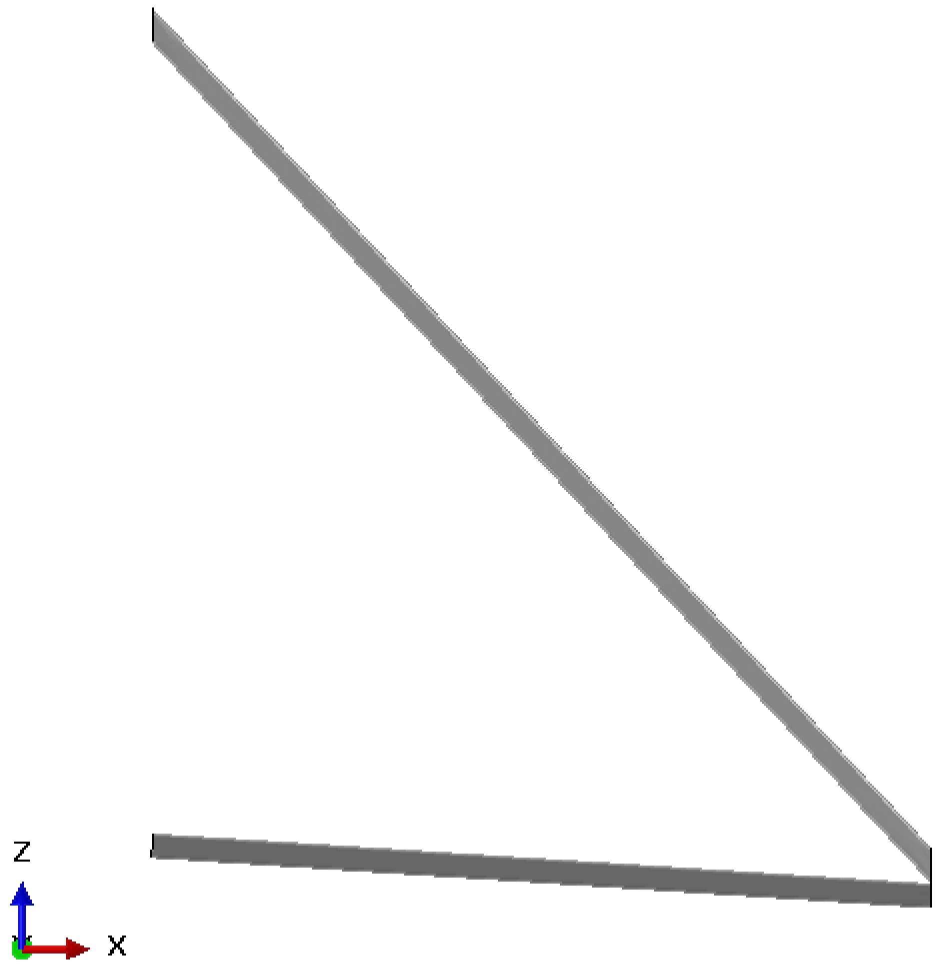
- maksimalne glavne deformacije



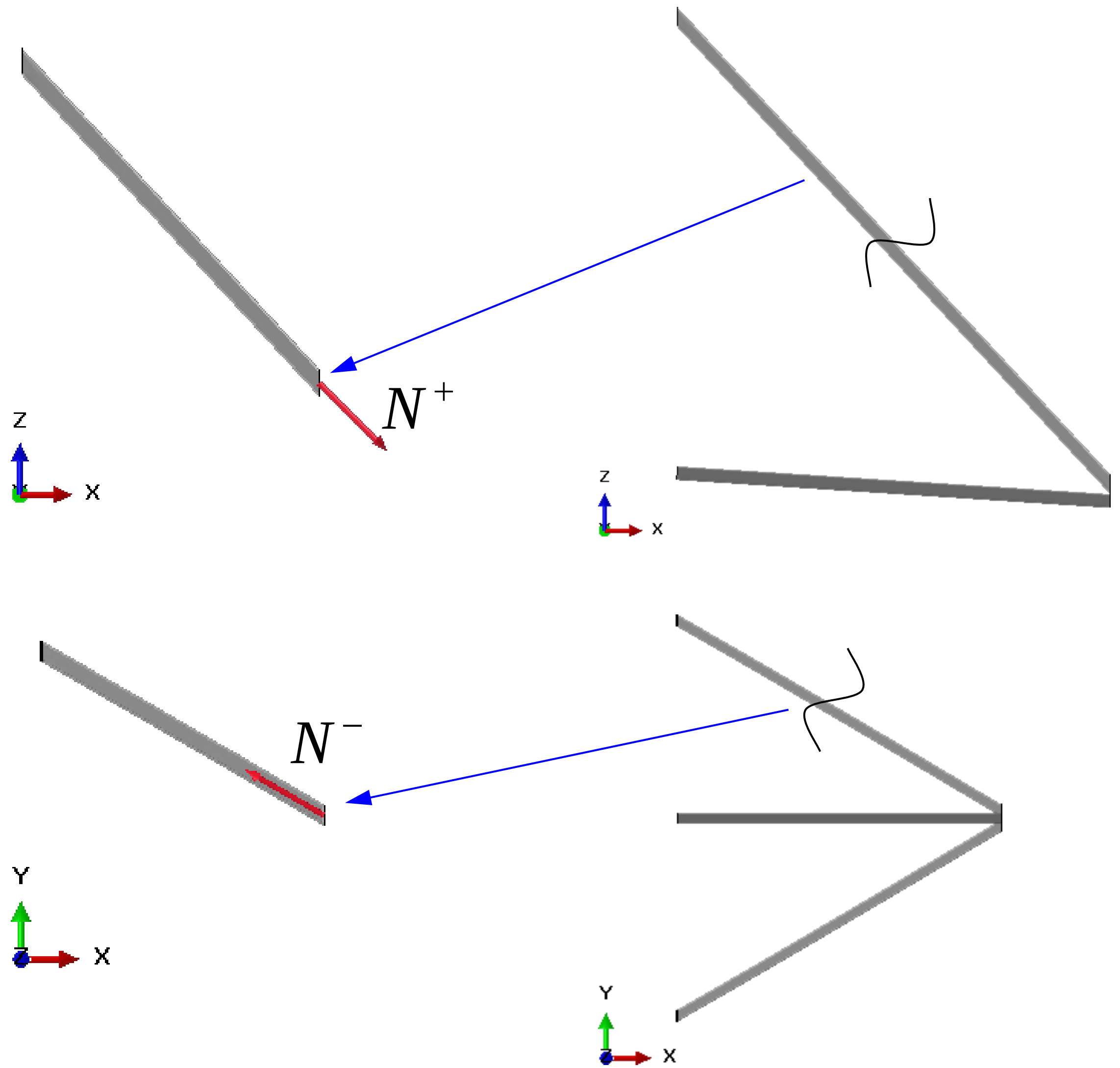
- Mises-ova primerjalna napetost



- deformirana konstrukcija (10x povečava)



- notranje sile v palicah

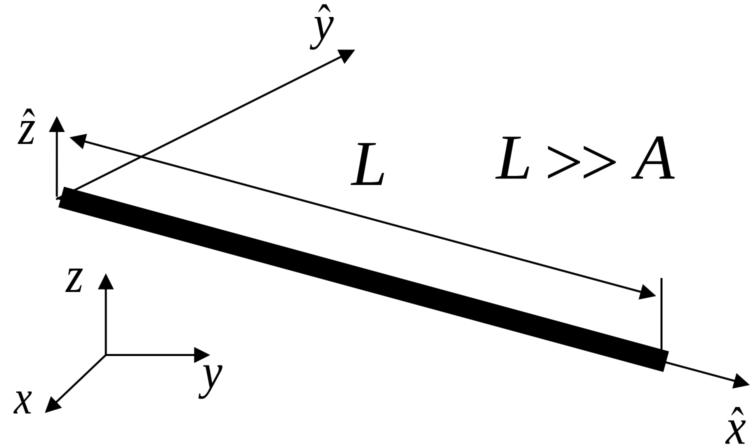


- kdaj lahko mehanski problem obravnavamo kot <u>paličje</u>?
  - da lahko problem obravnavamo kot <u>paličje</u>, mora biti izpolnjeno:

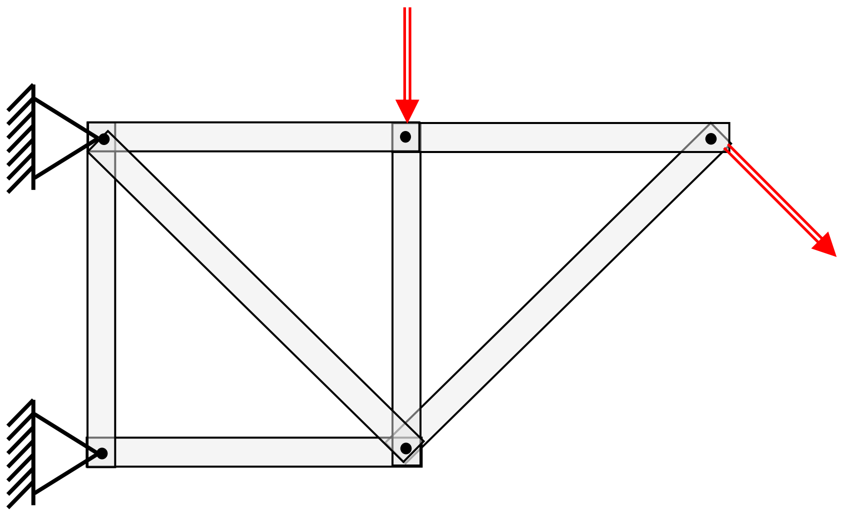
1) konstrukcijski element, imenovan palica, prenaša predvsem osno obremenitev

2) homogen, izotropen material

3) prerez palice je majhen glede na njeno dolžino

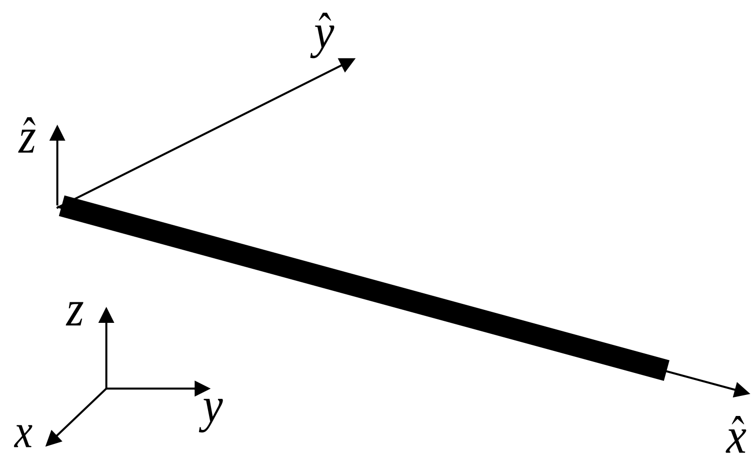


4) obremenjene smejo biti le povezave med palicami, pri čemer mora biti obremenitev točkovna



- od nič različna komponenta napetostnega tenzorja v <u>lokalnem Kartezijevem koordinatnem sistemu</u> palice je samo σ_{xx}

$$\sigma_{ij} = \begin{Bmatrix} \sigma_{\hat{x}\hat{x}} & \sigma_{\hat{x}\hat{y}} = 0 & \sigma_{\hat{x}\hat{z}} = 0 \\ \sigma_{\hat{x}\hat{y}} = 0 & \sigma_{\hat{y}\hat{y}} = 0 & \sigma_{\hat{y}\hat{z}} = 0 \\ \sigma_{\hat{x}\hat{z}} = 0 & \sigma_{\hat{y}\hat{z}} = 0 & \sigma_{\hat{z}\hat{z}} = 0 \end{Bmatrix}$$



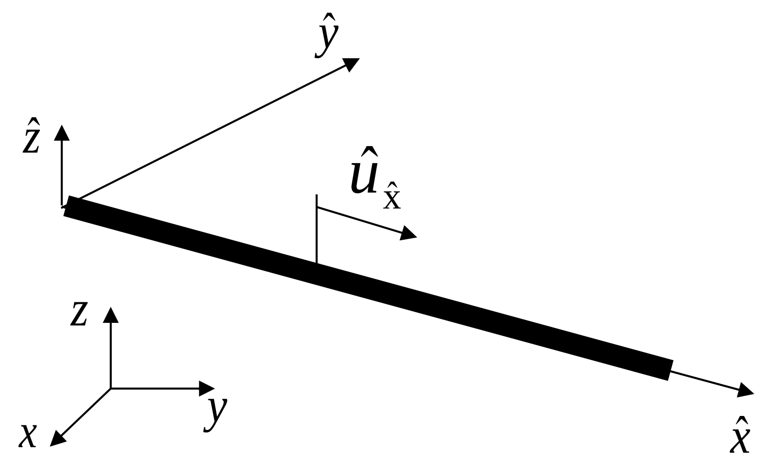
- komponente deformacijskega tenzorja lahko v lokalnem Kartezijevem koordinatnem sistemu palice zapišemo v odvisnosti od pomika v smeri x koordinatne osi

$$\varepsilon_{\hat{x}\hat{x}} = \frac{\partial \hat{u}_{\hat{x}}}{\partial \hat{x}}$$

$$\varepsilon_{\hat{x}\hat{y}} = 0 \;,\; \varepsilon_{\hat{x}\hat{z}} = 0 \;,\; \varepsilon_{\hat{y}\hat{z}} = 0$$

$$\varepsilon_{\hat{y}\hat{y}} = ?$$

$$\varepsilon_{\hat{z}\hat{z}} = ?$$



- za homogeni, izotropni, linearno elastični material, lahko iz zveze med napetostmi in deformacijami, ki jo definira Hookov zakon, izračunamo komponenti deformacijskega tenzorja ε_{yy} in ε_{zz}

$$\sigma_{\hat{x}\hat{x}} = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{\hat{x}\hat{x}} + \nu\varepsilon_{\hat{y}\hat{y}} + \nu\varepsilon_{\hat{z}\hat{z}}]$$

$$\sigma_{\hat{y}\hat{y}} = 0 = \frac{E}{(1+\nu)(1-2\nu)}[\nu\varepsilon_{\hat{x}\hat{x}} + (1-\nu)\varepsilon_{\hat{y}\hat{y}} + \nu\varepsilon_{\hat{z}\hat{z}}]$$

$$\sigma_{\hat{z}\hat{z}} = 0 = \frac{E}{(1+\nu)(1-2\nu)}[\nu\varepsilon_{\hat{x}\hat{x}} + \nu\varepsilon_{\hat{y}\hat{y}} + (1-\nu)\varepsilon_{\hat{z}\hat{z}}]$$

$$\varepsilon_{\hat{y}\hat{y}} = \varepsilon_{\hat{z}\hat{z}}$$

$$\sigma_{\hat{y}\hat{y}} = 0 = \frac{E}{(1+\nu)(1-2\nu)}[\nu\varepsilon_{\hat{x}\hat{x}} + (1-\nu)\varepsilon_{\hat{y}\hat{y}} + \nu\varepsilon_{\hat{z}\hat{z}}] \Rightarrow \varepsilon_{\hat{y}\hat{y}} = \varepsilon_{\hat{z}\hat{z}} = -\nu\varepsilon_{\hat{x}\hat{x}}$$

$$\sigma_{\hat{x}\hat{x}} = E\,\varepsilon_{\hat{x}\hat{x}}$$

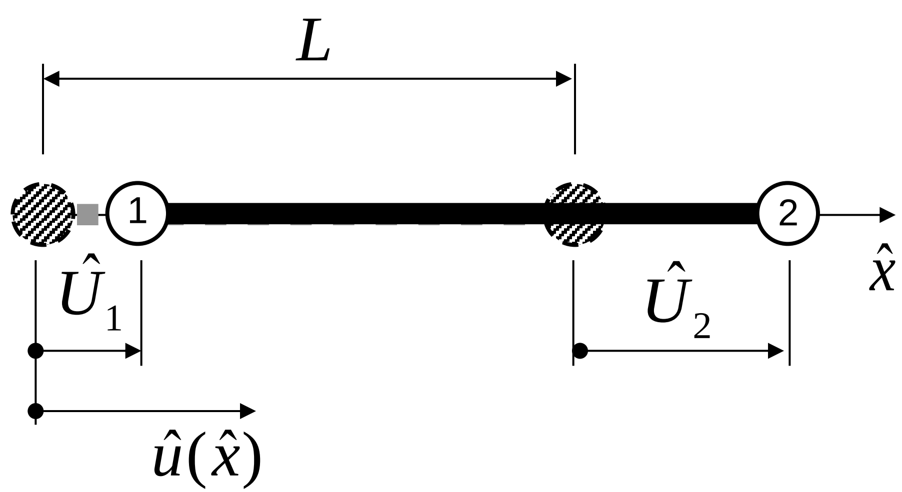
• matrični zapis enačbe KE za linearno elastično statično obremenjeno palico

• funkcijski zapis pomika v lokalnem Kartezijevem koordinatnem sistemu palice

$$\hat{U}_1 = u_{\hat{x}}(\hat{x} = 0)$$

$$\hat{U}_2 = u_{\hat{x}}(\hat{x} = L)$$

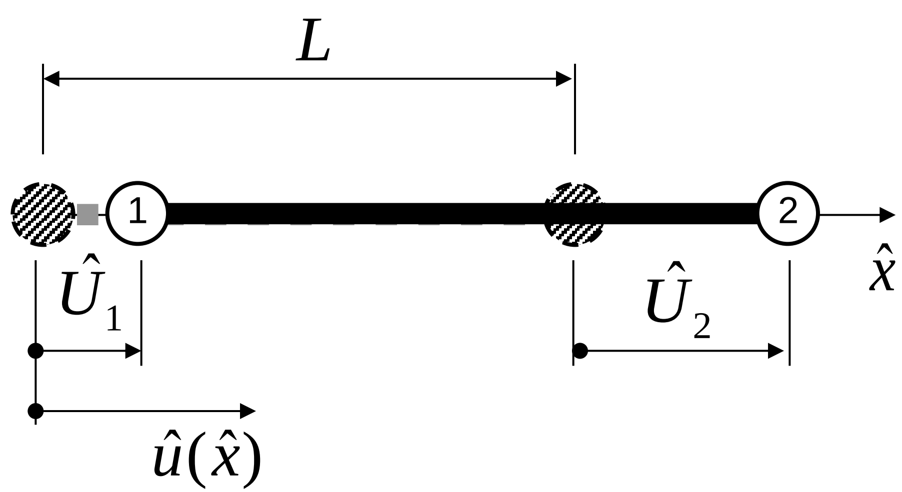
$$u_{\hat{x}}(\hat{x}) = \hat{U}_1\left(1 - \frac{\hat{x}}{L}\right) + \hat{U}_2\left(\frac{\hat{x}}{L}\right)$$



• izračun osne deformacije

$$\hat{u}_{\hat{x}}(\hat{x}) = \hat{U}_1\left(1-\frac{\hat{x}}{L}\right)+\hat{U}_2\left(\frac{\hat{x}}{L}\right)$$

$$\varepsilon_{\hat{x}\hat{x}} = \frac{\partial \hat{u}_{\hat{x}}}{\partial \hat{x}} = \frac{\hat{U}_2 - \hat{U}_1}{L}$$

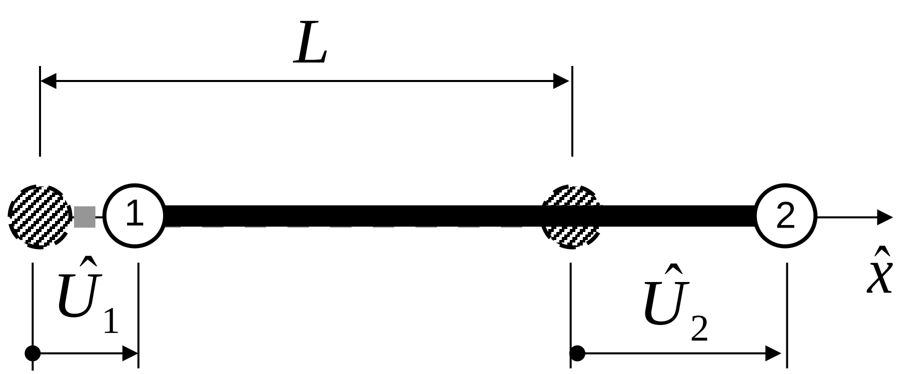


- izračun notranje osne sile

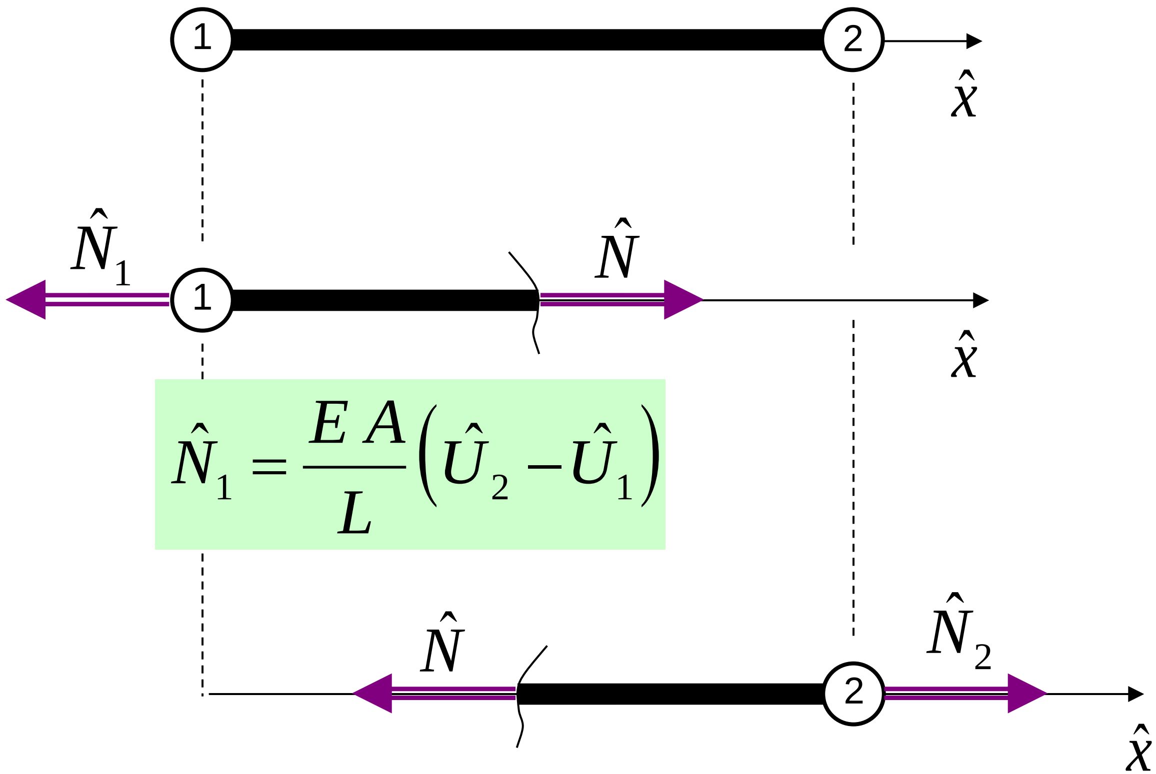
$$\varepsilon_{\hat{x}\hat{x}} = \frac{\partial u_{\hat{x}}}{\partial \hat{x}} = \frac{\left(\hat{U}_2 - \hat{U}_1\right)}{L}$$

$$\sigma_{\hat{x}\hat{x}} = E\,\varepsilon_{\hat{x}\hat{x}} = E\,\frac{\left(\hat{U}_2 - \hat{U}_1\right)}{L}$$

$$\sigma_{\hat{x}\hat{x}} = \frac{\hat{N}}{A} = E\,\frac{\left(\hat{U}_2 - \hat{U}_1\right)}{L} \Rightarrow \hat{N} = \frac{E\,A}{L}\left(\hat{U}_2 - \hat{U}_1\right)$$



- izračun notranje osne sile v vozlišču KE



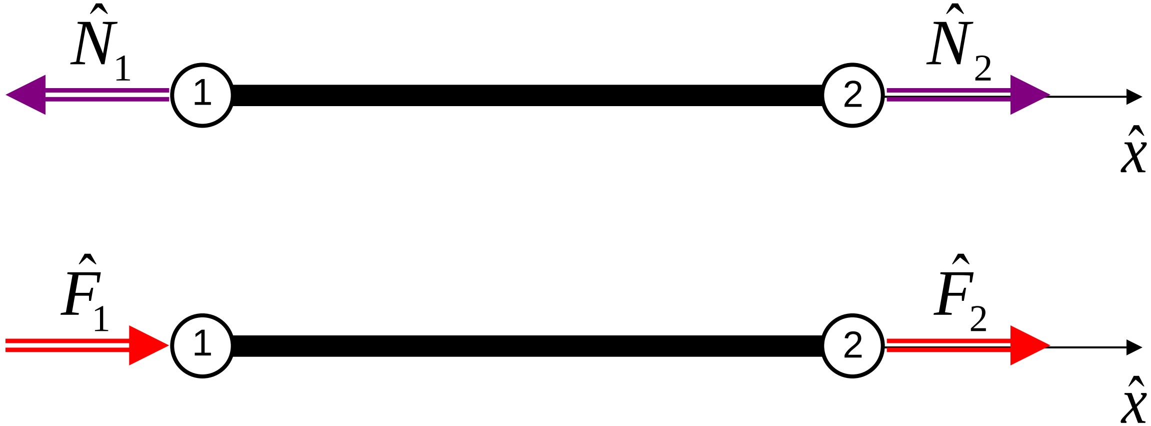
$$\hat{N}_1 = \frac{E\,A}{L}\left(\hat{U}_2 - \hat{U}_1\right)$$

$$\hat{N}_2 = \frac{E\,A}{L}\left(\hat{U}_2 - \hat{U}_1\right)$$

- izračun sile, ki deluje v vozlišču KE

$$\hat{F}_1 = -\hat{N}_1 = -\frac{E\,A}{L}\left(\hat{U}_2 - \hat{U}_1\right) = +\frac{E\,A}{L}\left(\hat{U}_1 - \hat{U}_2\right)$$

$$\hat{F}_2 = +\hat{N}_2 = +\frac{E\,A}{L}\left(\hat{U}_2 - \hat{U}_1\right) = -\frac{E\,A}{L}\left(\hat{U}_1 - \hat{U}_2\right)$$

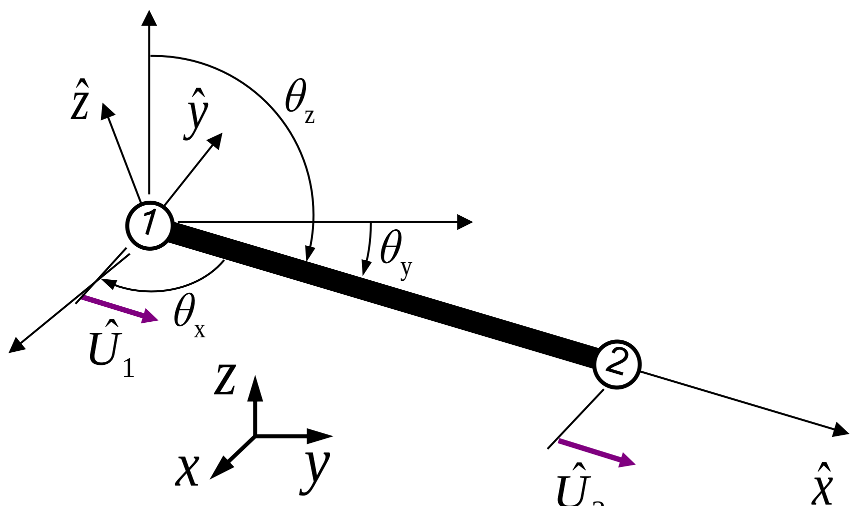


- zapis enačb KE v matrični obliki v lokalnem koordinatnem sistemu palice

$$\frac{E\,A}{L}\begin{bmatrix}+1 & -1\\ -1 & +1\end{bmatrix}\begin{Bmatrix}\hat{U}_1\\ \hat{U}_2\end{Bmatrix}=\begin{Bmatrix}\hat{F}_1\\ \hat{F}_2\end{Bmatrix}$$

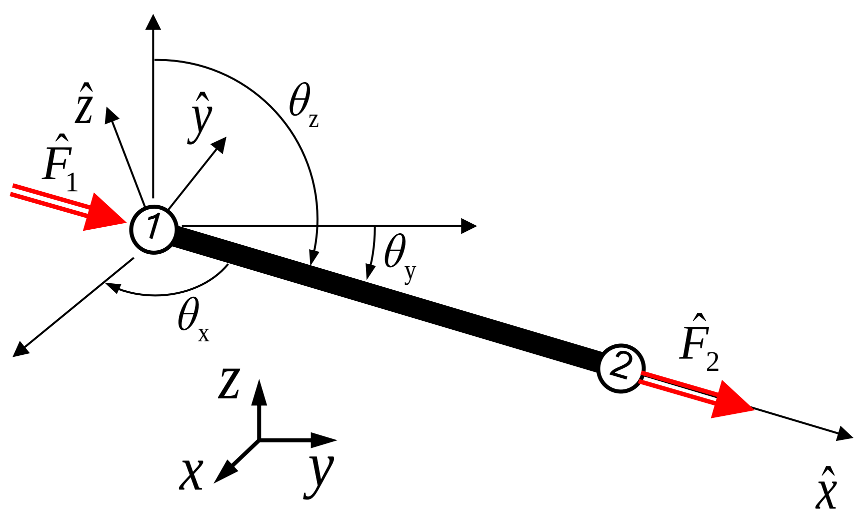
- transformacija pomikov iz lokalnega v globalni koordinatni sistem obravnavanega problema

$$\begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta_x & \cos\theta_y & \cos\theta_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta_x & \cos\theta_y & \cos\theta_z \end{bmatrix} \begin{Bmatrix} U_{1x} \\ U_{1y} \\ U_{1z} \\ U_{2x} \\ U_{2y} \\ U_{2z} \end{Bmatrix}$$



- transformacija sil iz <u>lokalnega</u> v <u>globalni</u> koordinatni sistem obravnavanega problema

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{F}_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta_x & \cos\theta_y & \cos\theta_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta_x & \cos\theta_y & \cos\theta_z \end{bmatrix} \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{1z} \\ F_{2x} \\ F_{2y} \\ F_{2z} \end{Bmatrix}$$



globalni k.s.

- matrični zapis enačbe KE za linearno elastični statično obremenjeni problem v globalnem koordinatnem sistemu

- za posamezni KE dobimo toliko enačb, kolikor ima KE prostostnih stopenj
- v vozlišču KE so neznane tri primarne veličine – pomiki, tako da ima posamezni KE $(3{*}N_v)$ prostostnih stopenj

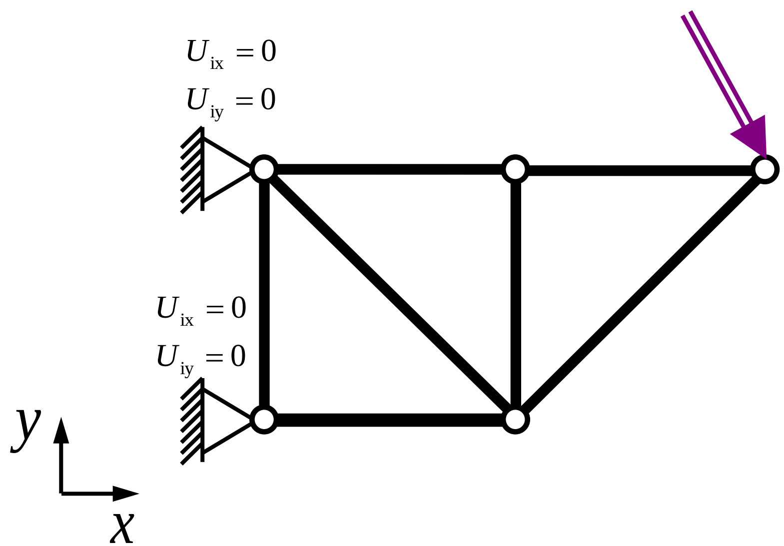
$$[K]_e \{U\}_e = \{F\}_e$$

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & \cdots & K_{1(3N_v)} \\ K_{21} & K_{22} & K_{23} & K_{24} & \cdots & K_{2(3N_v)} \\ K_{31} & K_{32} & K_{33} & K_{34} & \cdots & K_{3(3N_v)} \\ K_{41} & K_{42} & K_{43} & K_{44} & \cdots & K_{4(3N_v)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ K_{(3N_v)1} & K_{(3N_v)2} & K_{(3N_v)3} & K_{(3N_v)4} & \cdots & K_{(3N_v)(3N_v)} \end{bmatrix}_e \begin{Bmatrix} U_{1x} \\ U_{1y} \\ U_{1z} \\ U_{2x} \\ \vdots \\ U_{(3N_v)z} \end{Bmatrix}_e = \begin{Bmatrix} F_{1z} \\ F_{1y} \\ F_{1z} \\ F_{2x} \\ \vdots \\ F_{(3N_v)z} \end{Bmatrix}_e$$

- posamezni element vektorja $\{F\}_\mathrm{e}$ predstavlja v vozlišču KE delujočo vektorsko komponento sile v smeri določene koordinatne osi

- v primeru, da je <u>velikost vektorske komponente pomika</u> v smeri določene koordinatne osi v vozlišču KE <u>poznana</u>, velikost točkovne mehanske obremenitve v tej smeri <u>ni poznana</u>

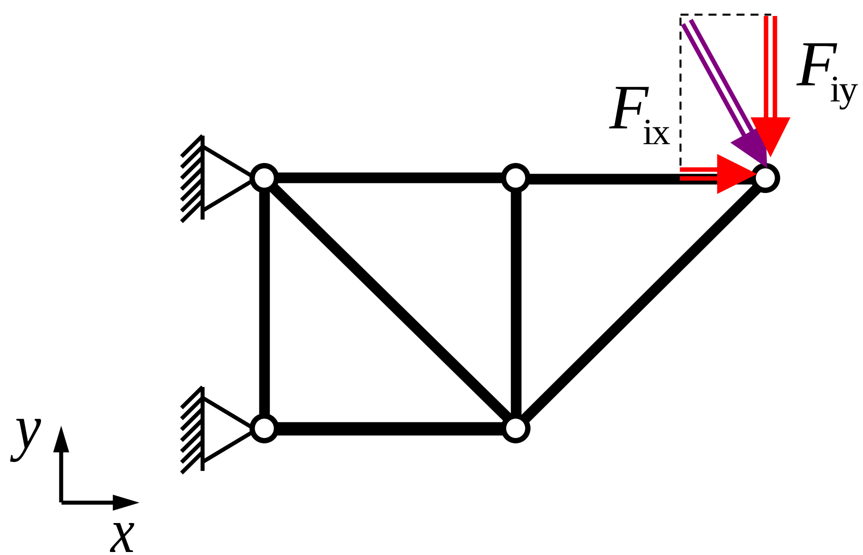
$$U_{\mathrm{ik}} = ✔ \Rightarrow \quad F_{\mathrm{ik}} = ? \,, \quad \mathrm{i} = 1,..,\mathrm{N_v} \;, \;\; \mathrm{k} = \mathrm{x,y,z}$$



- v primeru, da velikost komponente pomika v smeri določene koordinatne osi v vozlišču KE ni poznana, je velikost točkovne mehanske obremenitve v tej smeri možno izračunati

$$U_{ik} = ? \quad \Rightarrow \quad F_{ik} = \checkmark, \quad i = 1,.., N_v \ , \ k = x, y, z$$

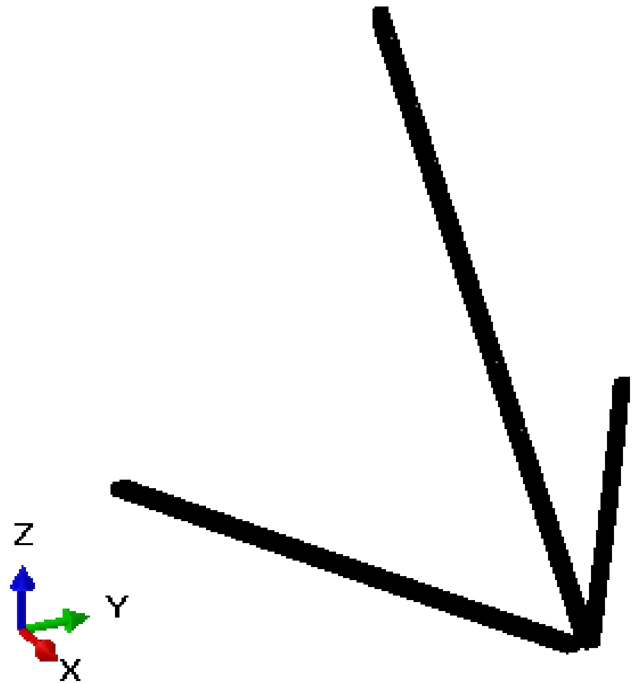
- točkovna obremenitev je vezana na vozlišče paličnega KE, ki leži na ograji obravnavnega območja KE

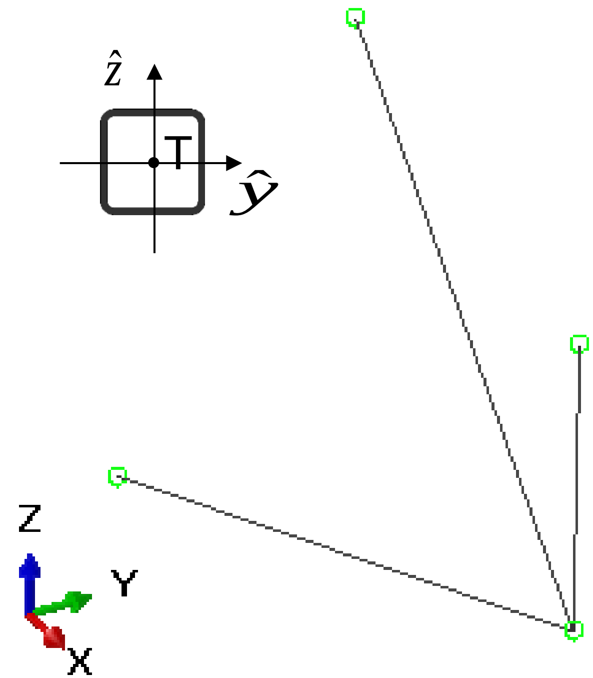


- primer reševanja <u>palične konstrukcije</u> z MKE

<u>3D KE:</u>
22000 KE (6 pl., 8 vozl.)
42000 vozlišč
126000 enačb

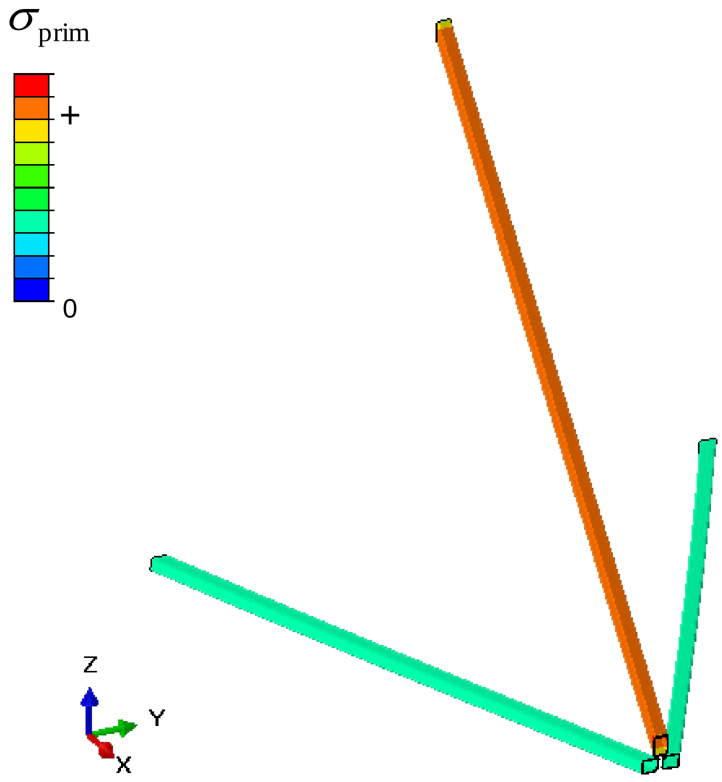
<u>1D palični KE:</u>
3 KE (2 vozliščni)
4 vozlišč
12 enačb



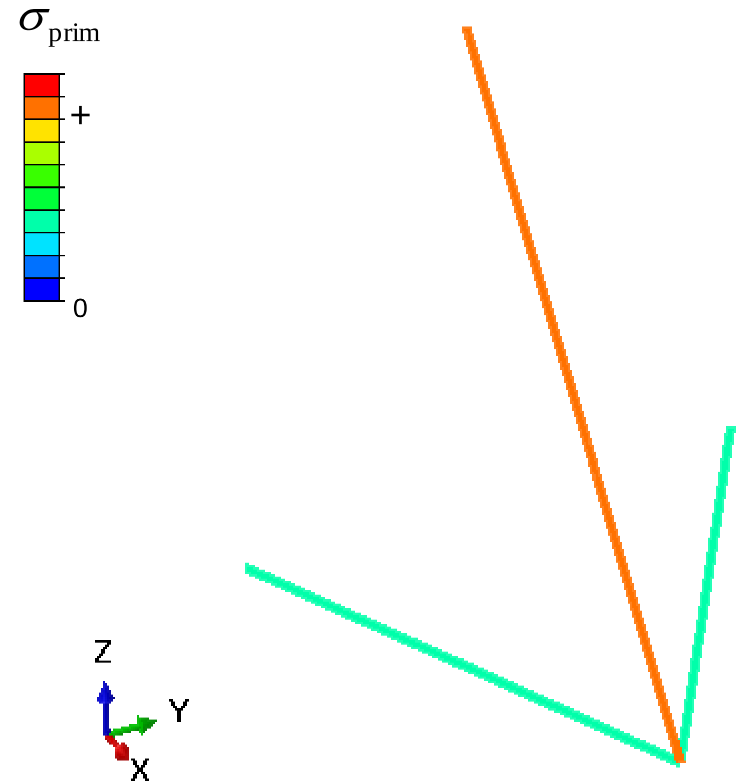


- primerjava Mises-ove primerjalne napetosti: 3D KE ⟷ 1D palični KE

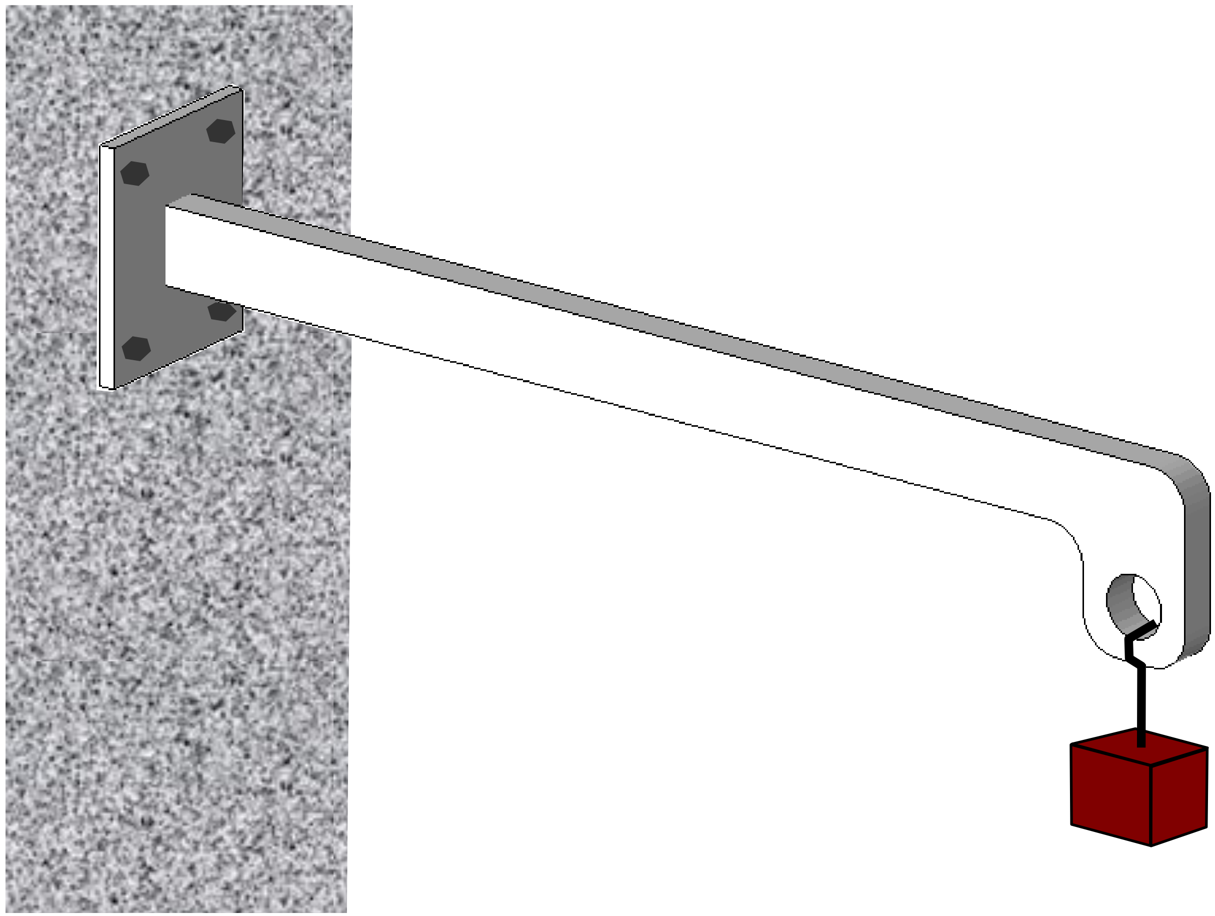
3D KE



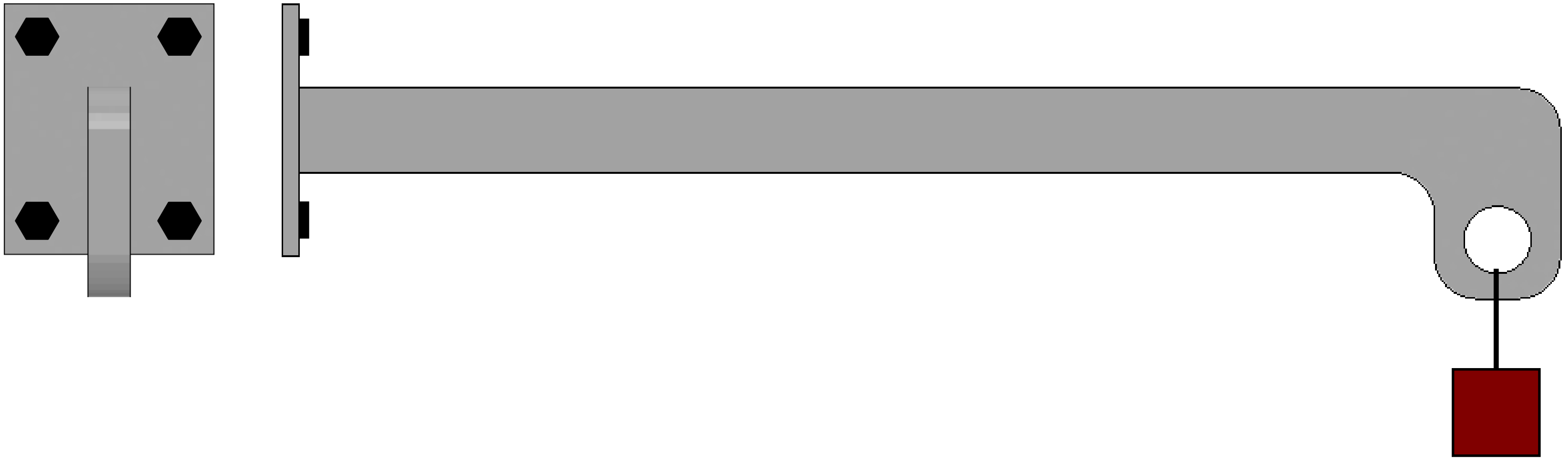
1D KE



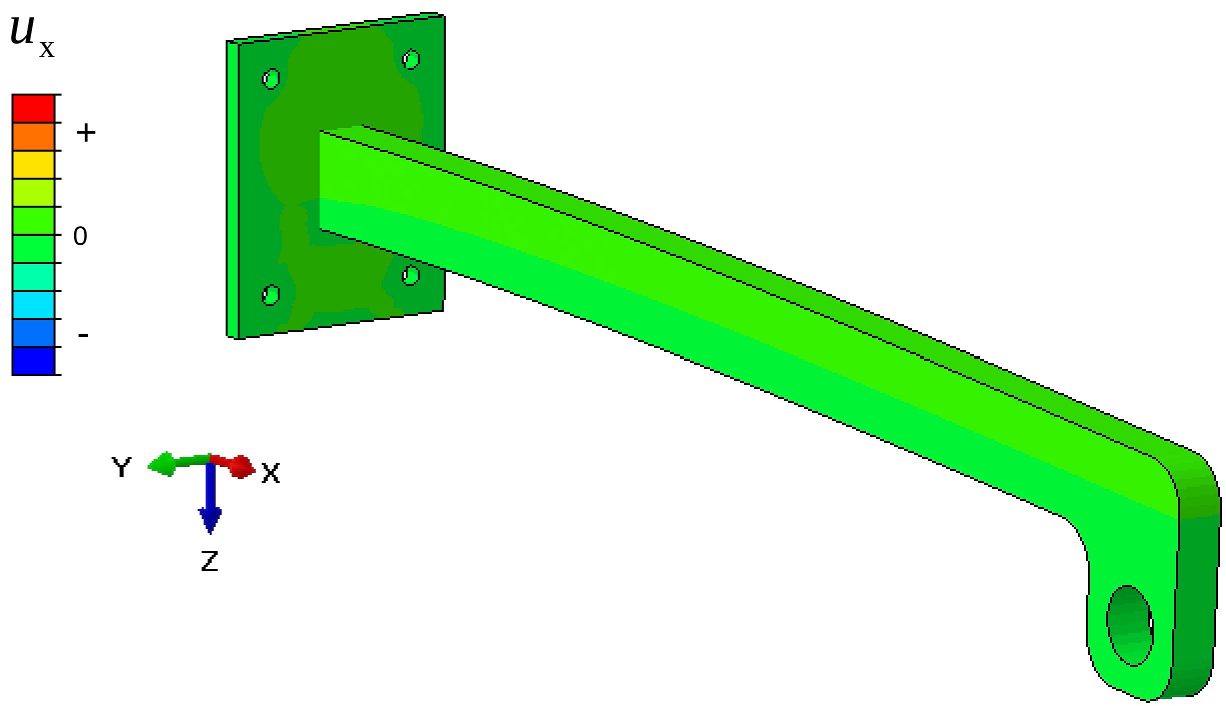
- primer reševanja volumskega mehanskega problema z MKE



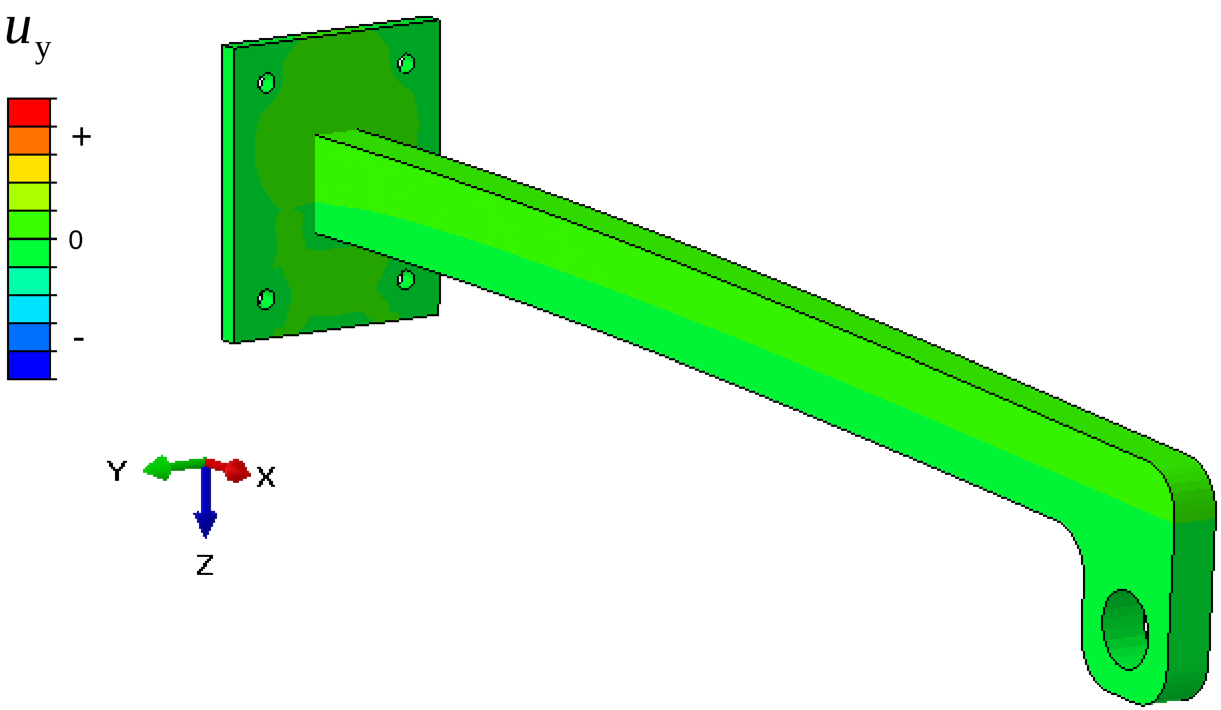
- primer reševanja volumskega mehanskega problema z MKE



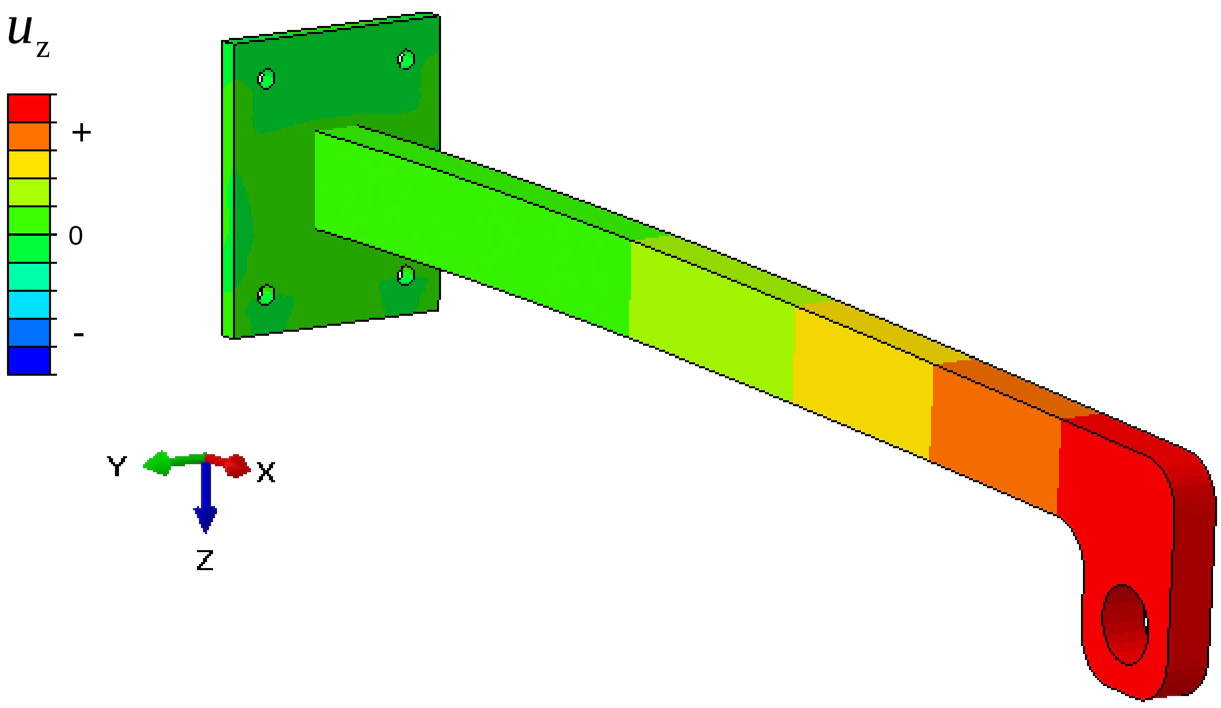
- pomiki v Kartezijevem koordinatnem sistemu



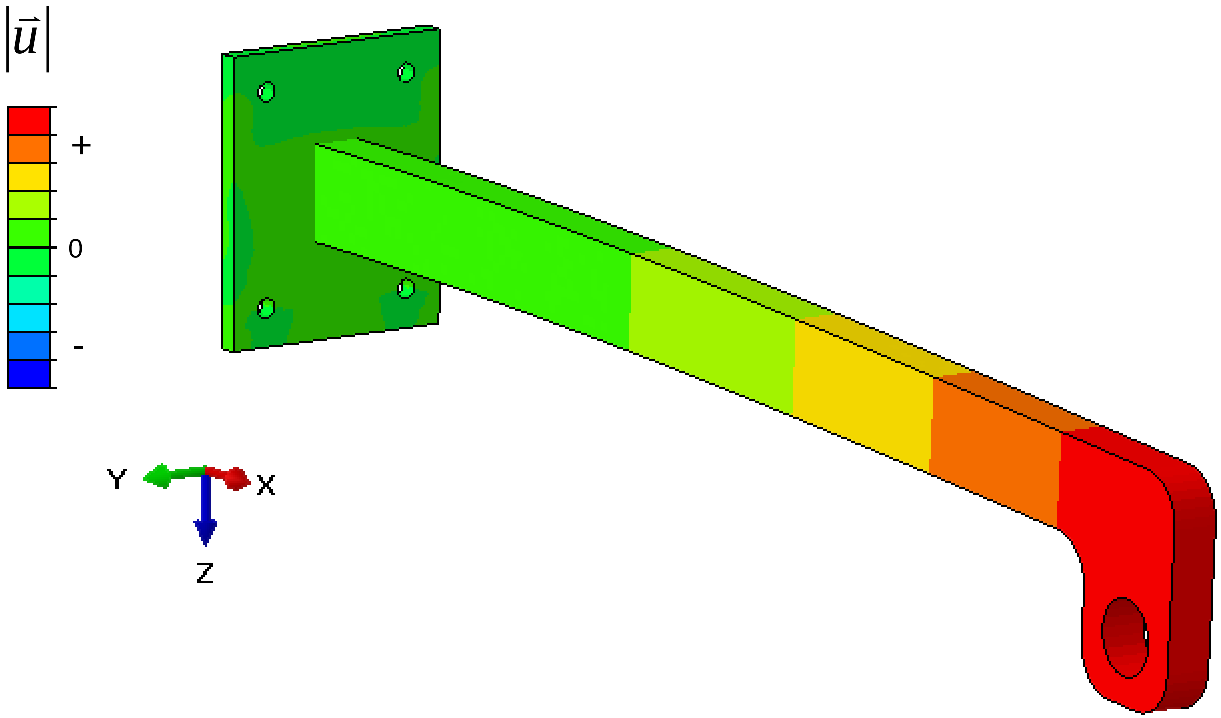
- pomiki v Kartezijevem koordinatnem sistemu



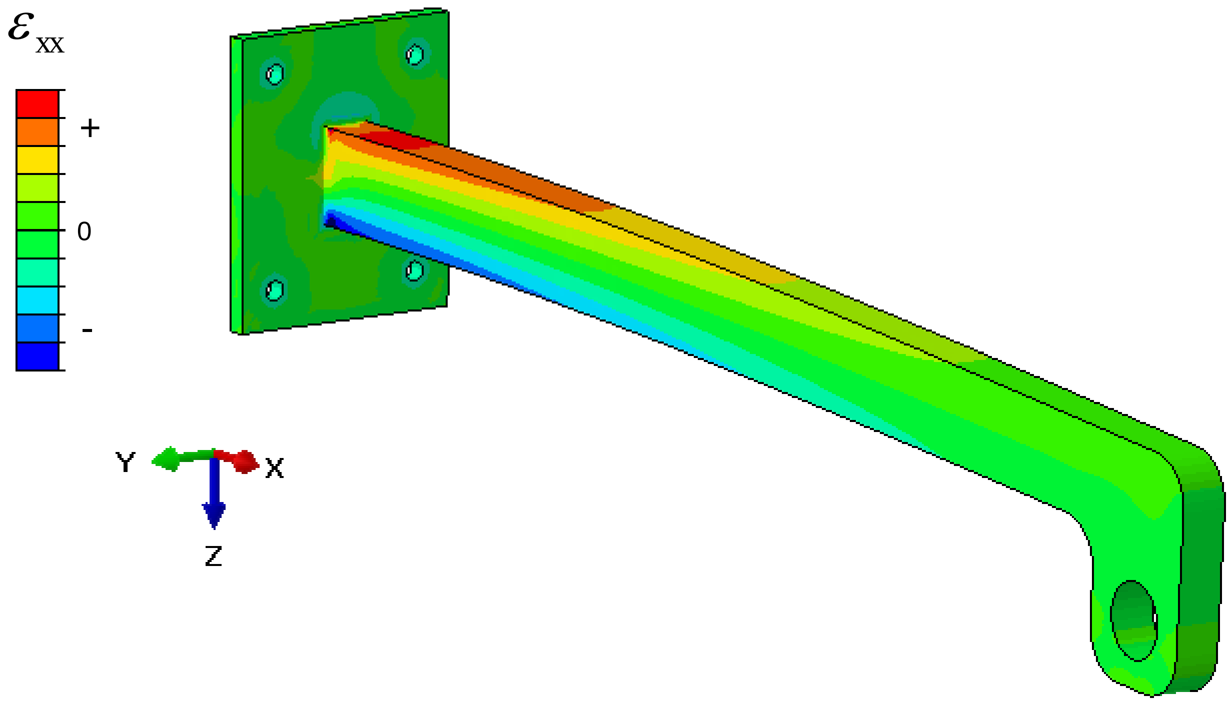
- pomiki v Kartezijevem koordinatnem sistemu



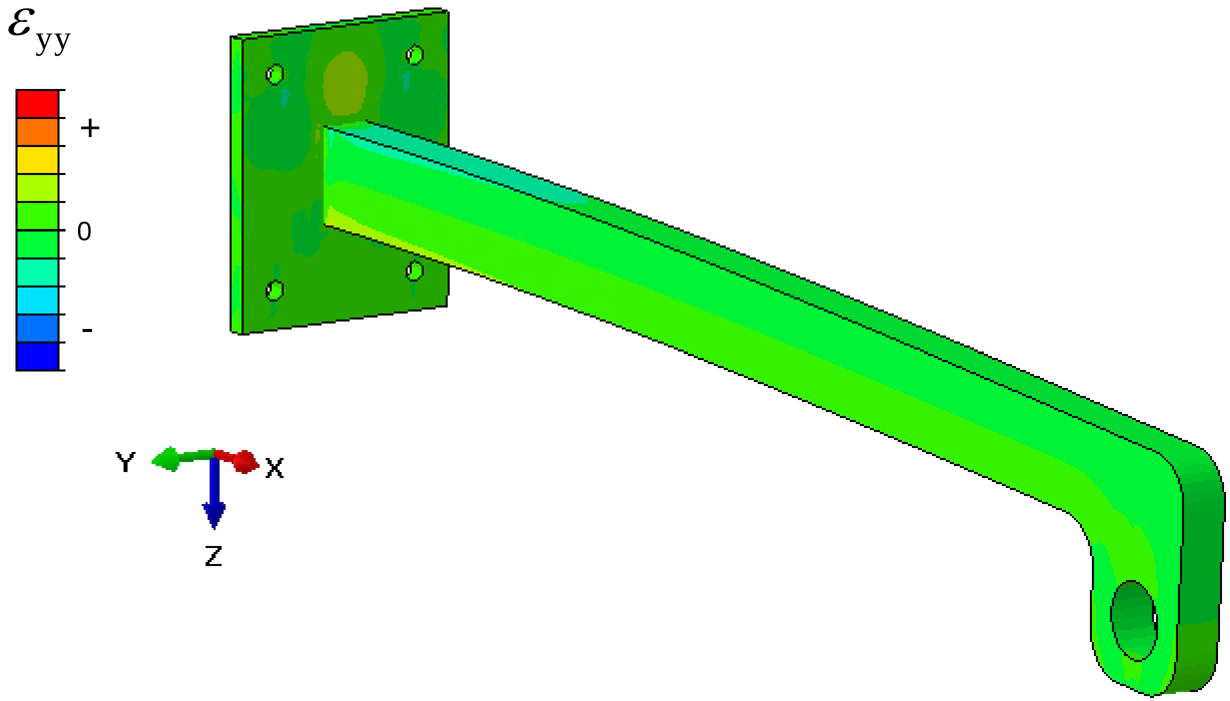
• pomiki v Kartezijevem koordinatnem sistemu



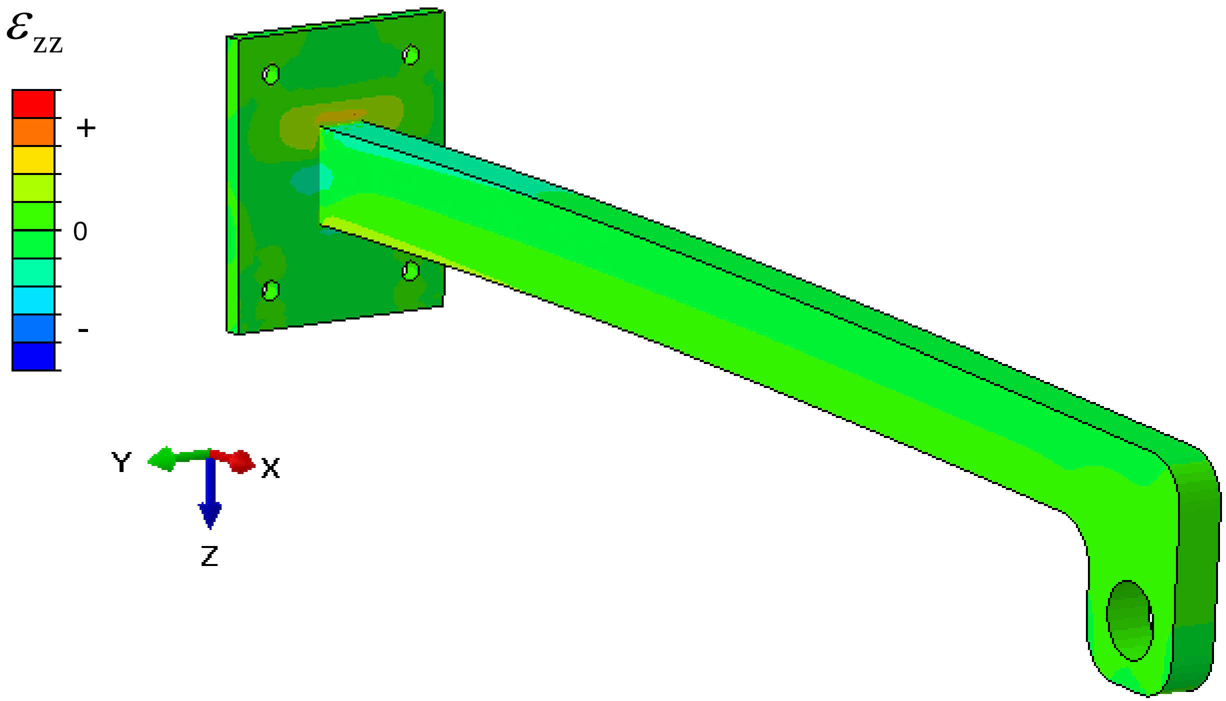
• deformacije v Kartezijevem koordinatnem sistemu



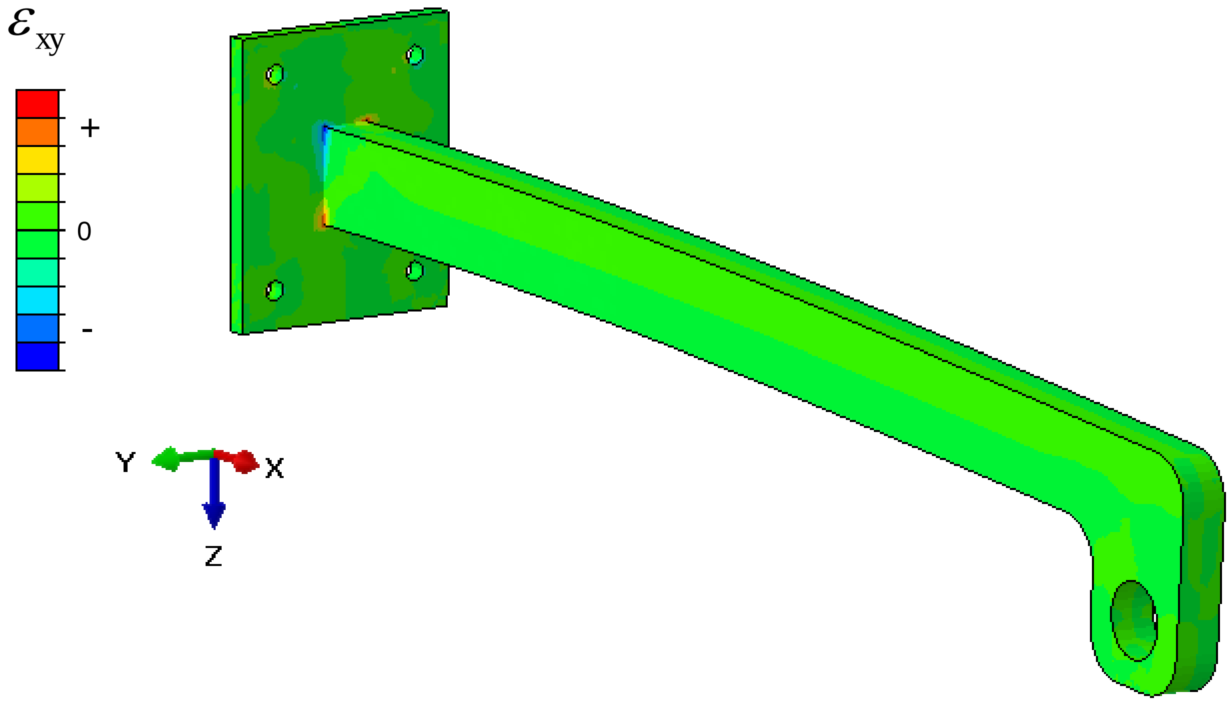
- deformacije v Kartezijevem koordinatnem sistemu



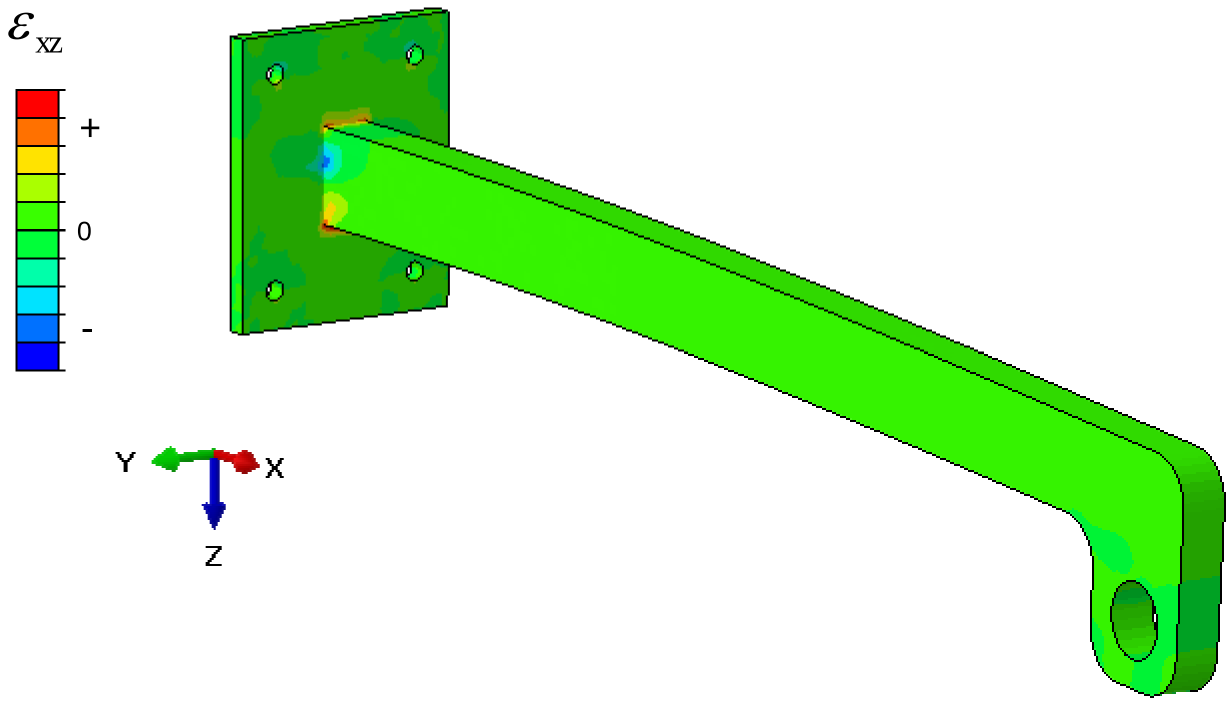
• deformacije v Kartezijevem koordinatnem sistemu



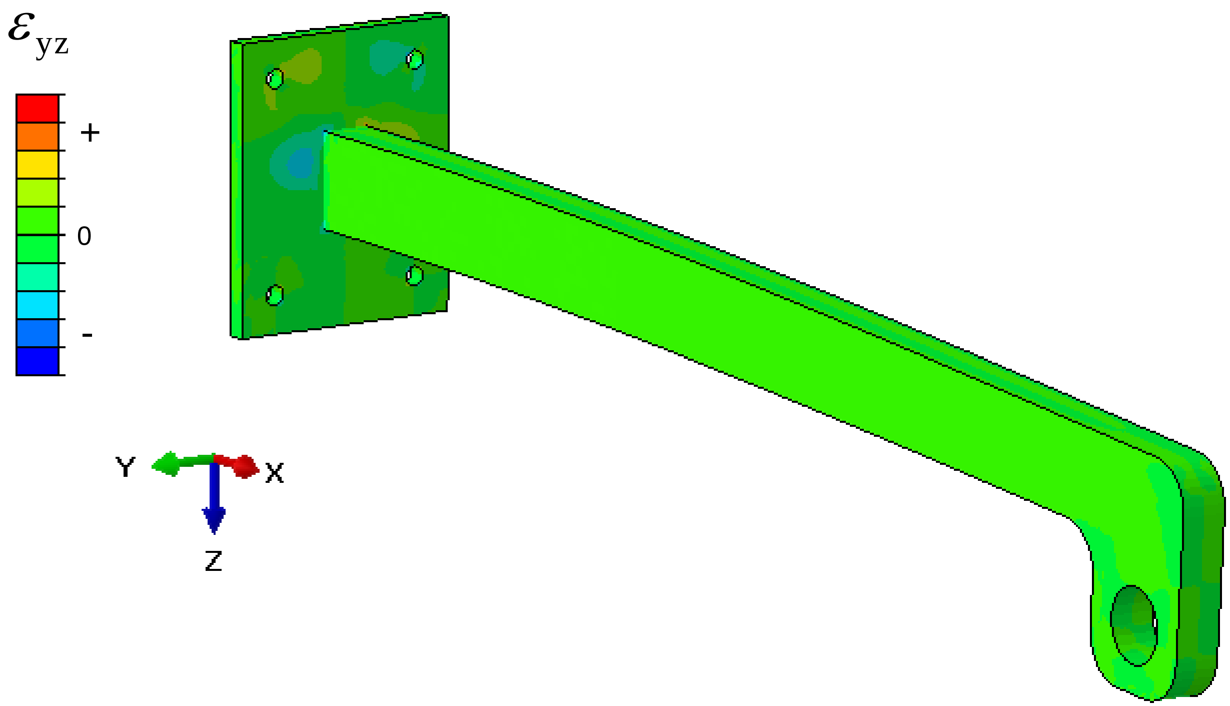
- deformacije v Kartezijevem koordinatnem sistemu



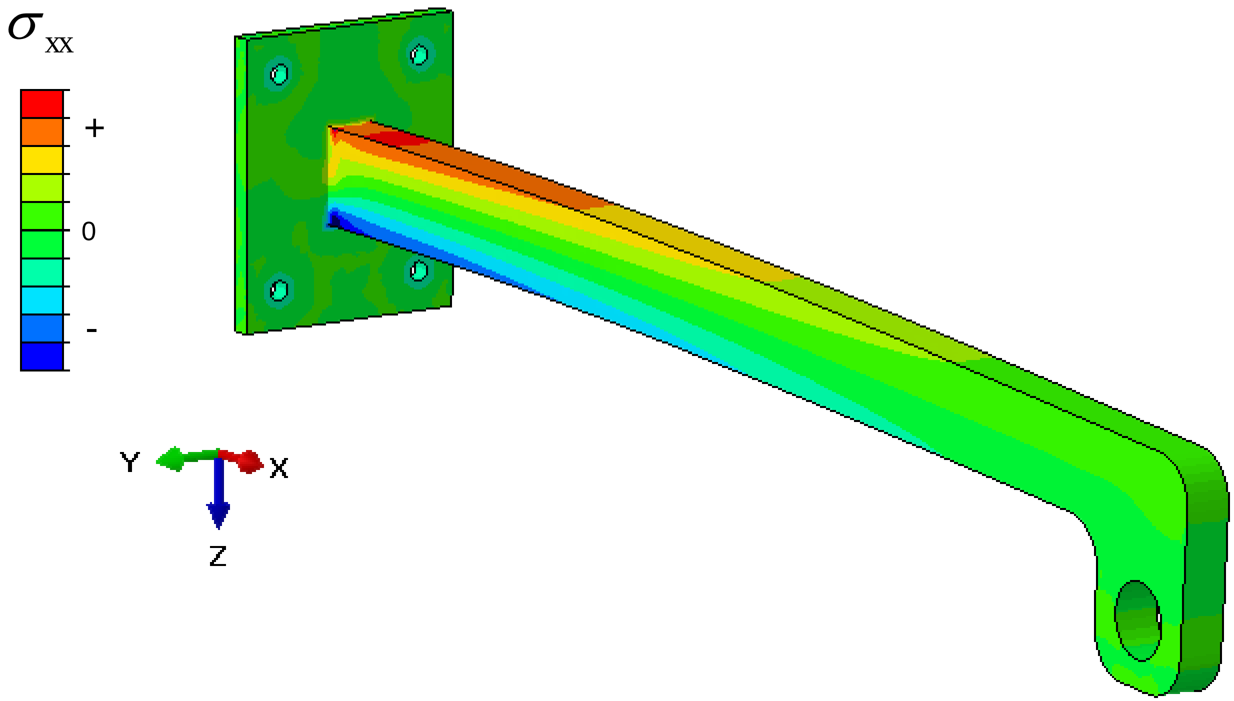
- deformacije v Kartezijevem koordinatnem sistemu



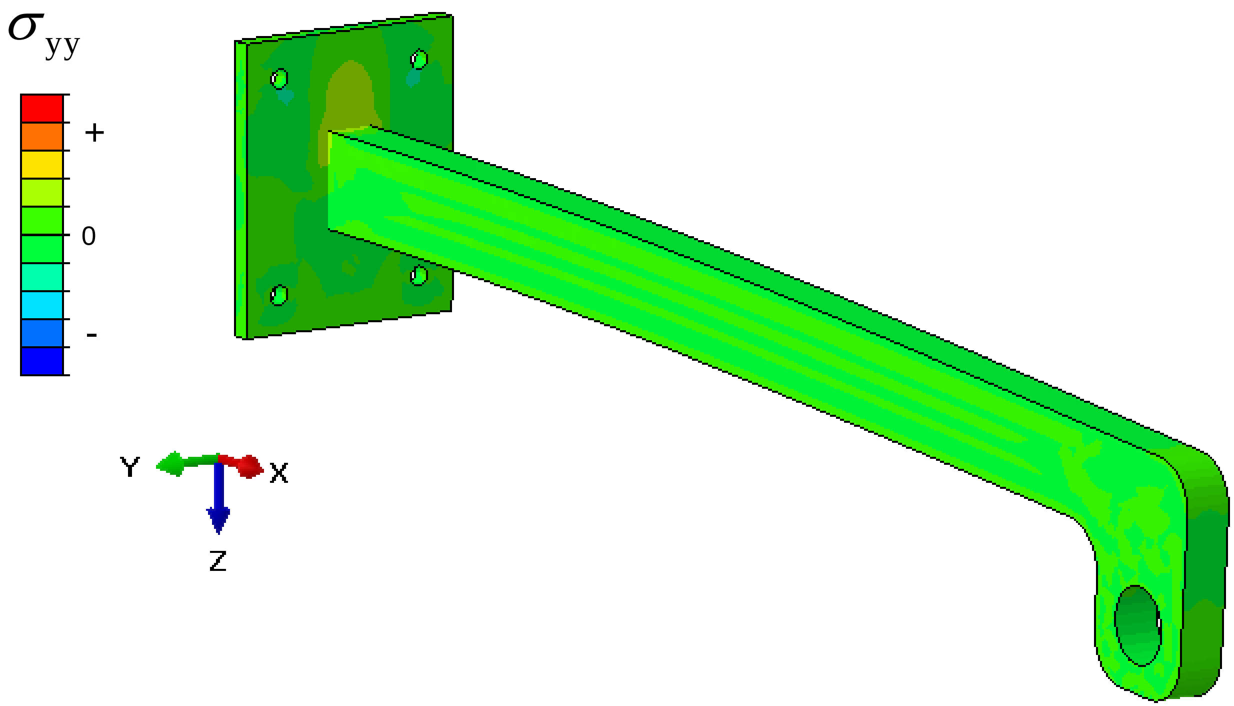
- deformacije v Kartezijevem koordinatnem sistemu



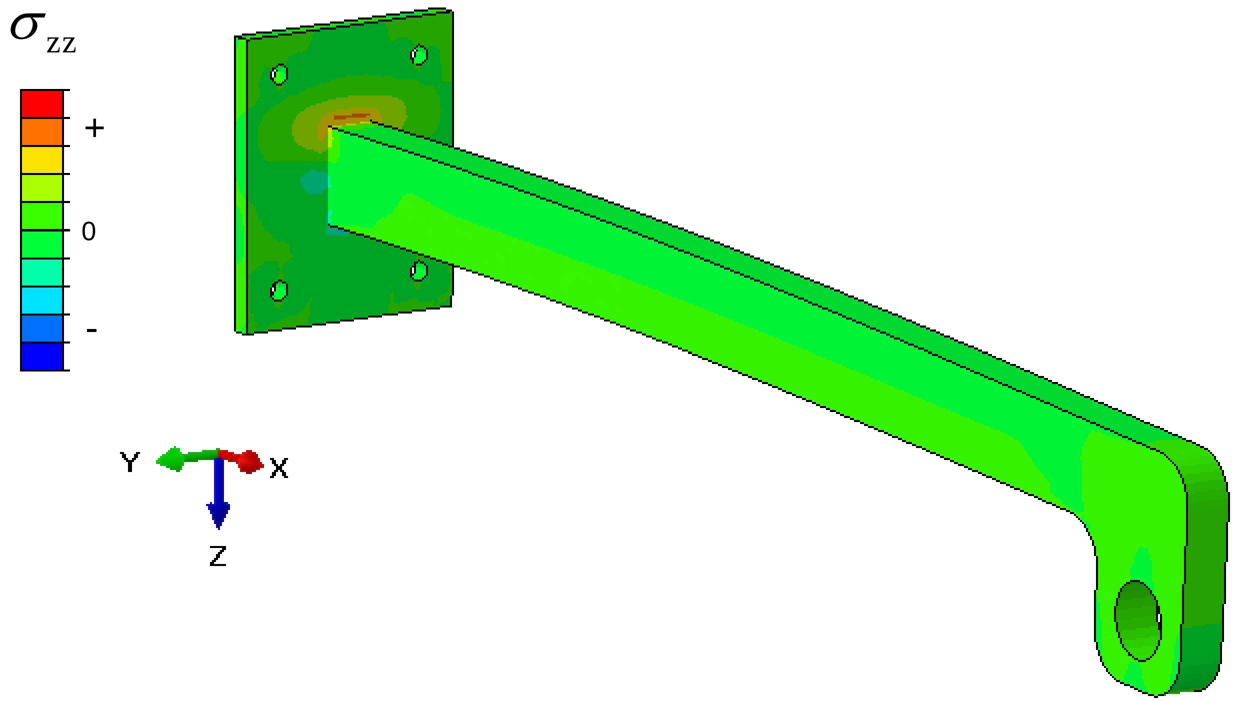
- napetosti v Kartezijevem koordinatnem sistemu



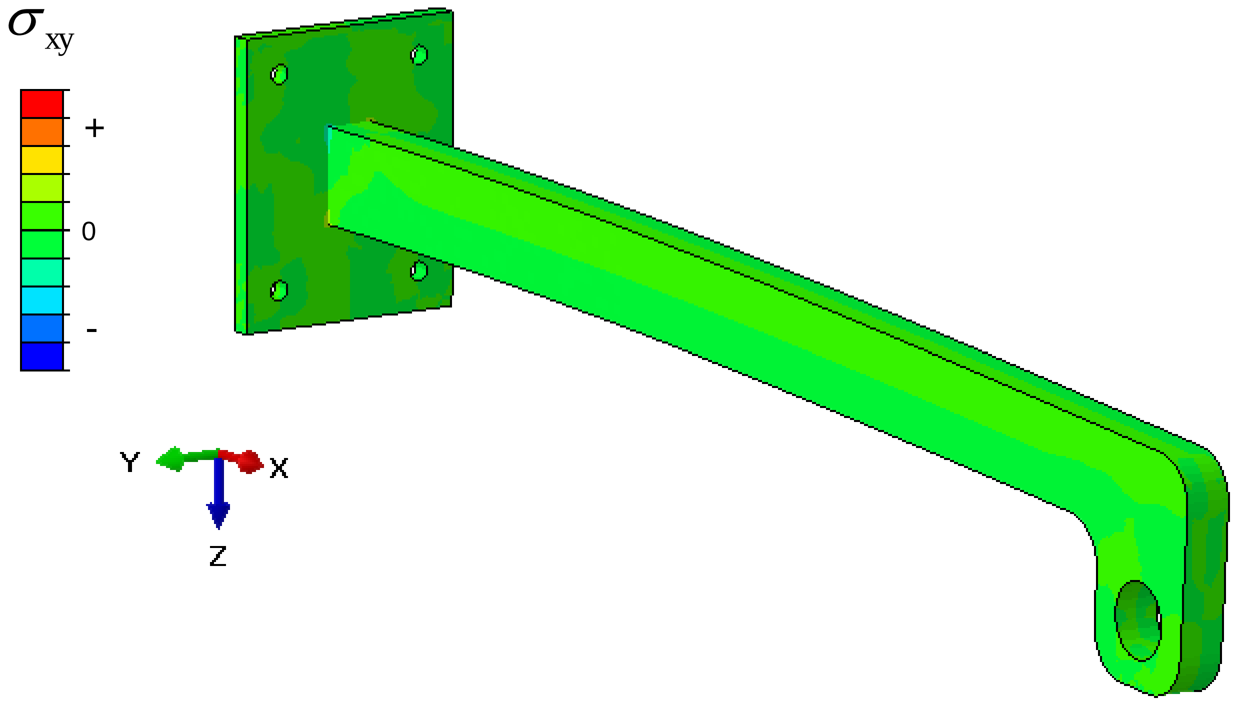
- napetosti v Kartezijevem koordinatnem sistemu



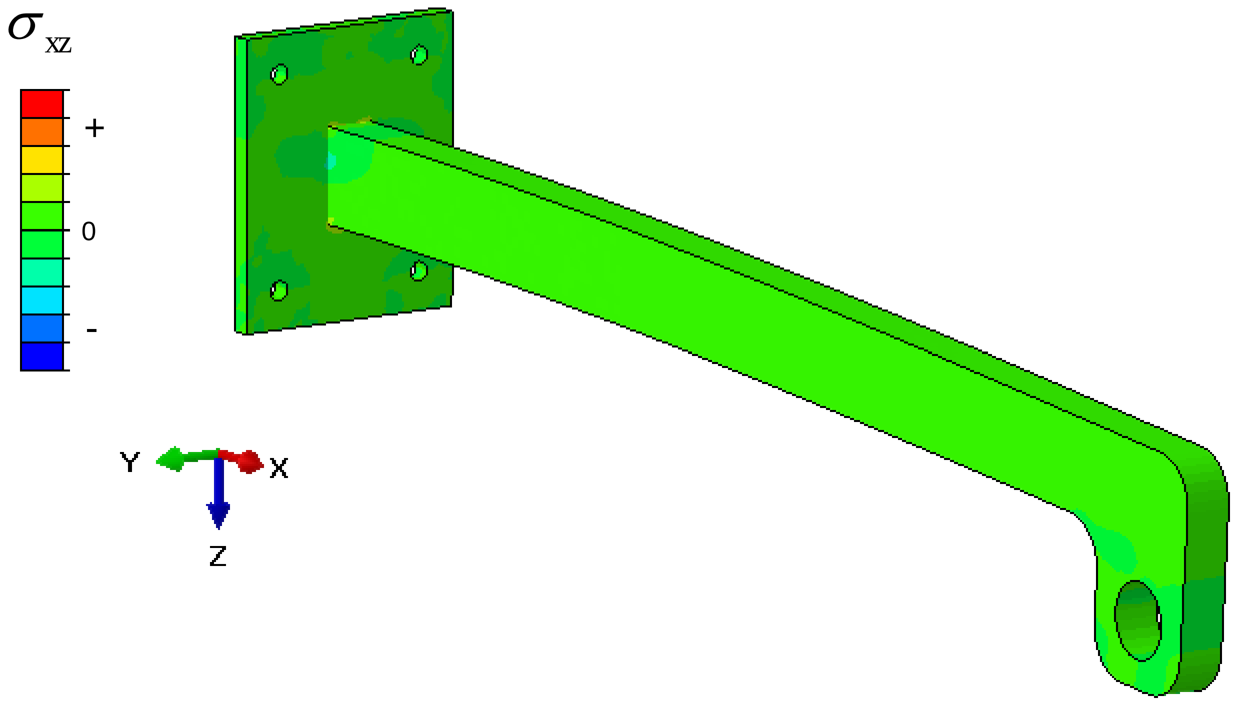
- napetosti v Kartezijevem koordinatnem sistemu



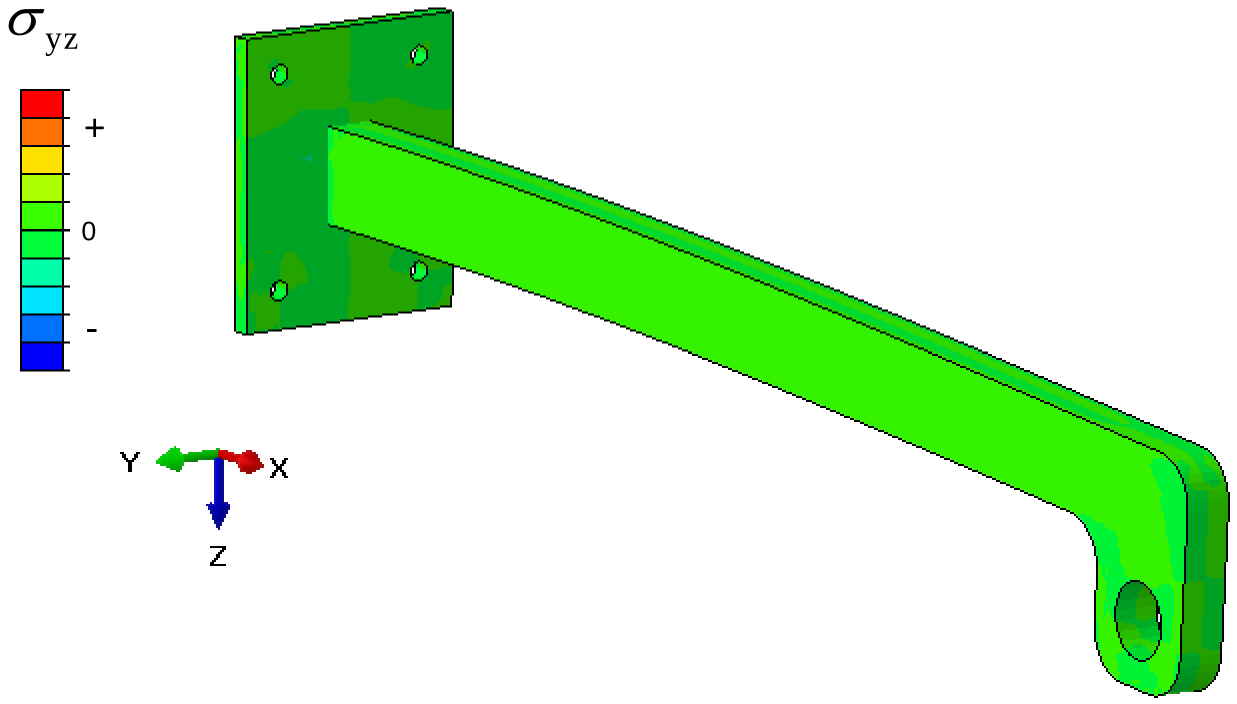
- napetosti v Kartezijevem koordinatnem sistemu



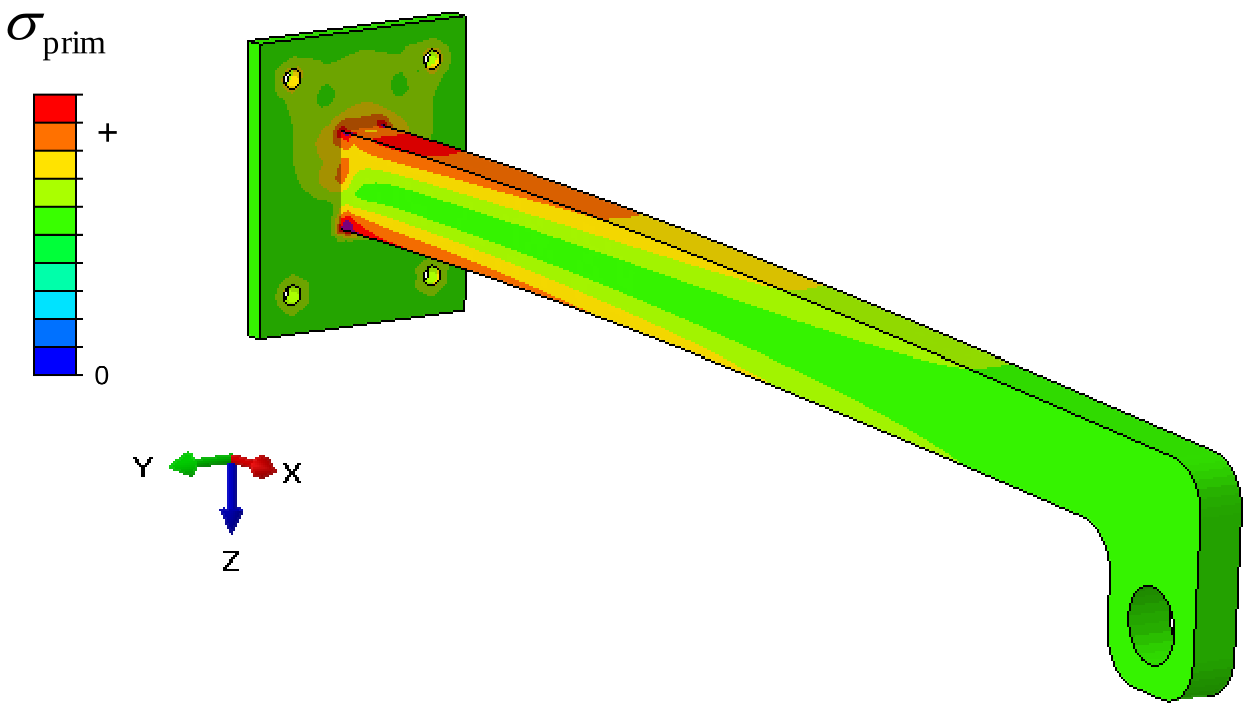
- napetosti v Kartezijevem koordinatnem sistemu



- napetosti v Kartezijevem koordinatnem sistemu



• Mises-ova primerjalna napetost



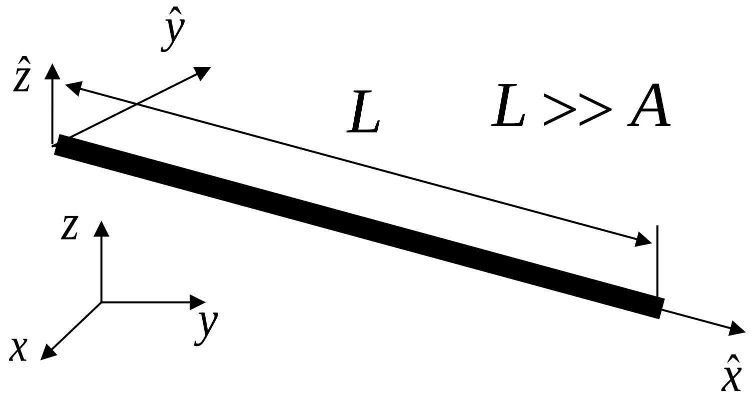
- kdaj lahko mehanski problem obravnavamo kot upogibno obremenjeni nosilec v ravnin ?

  - da lahko problem obravnavamo kot upogibno obremenjeni nosilec v ravnini (predpostavimo da je to ravnina (x,z)) mora biti izpolnjeno:

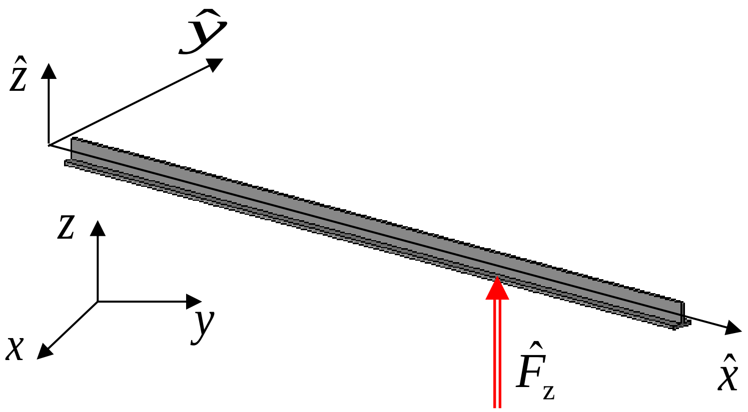
    1) konstrukcijski element, imenovan nosilec, je obremenjen predvsem upogibno

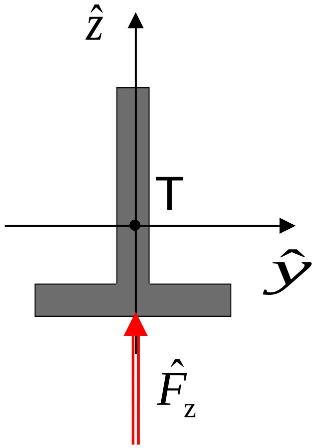
    2) homogen, izotropen material

    3) prerez nosilca je majhen glede na njegovo dolžino

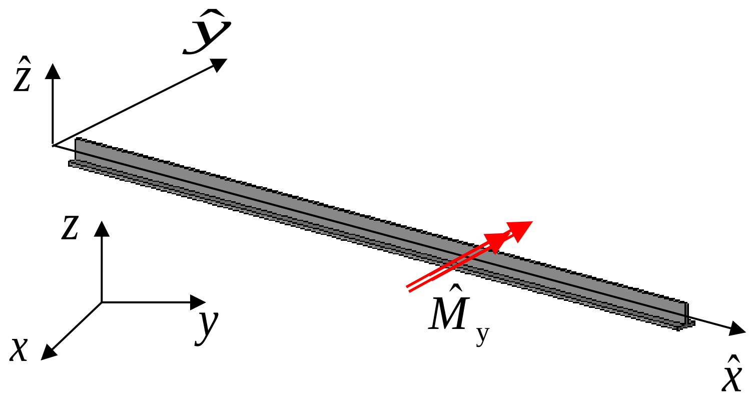


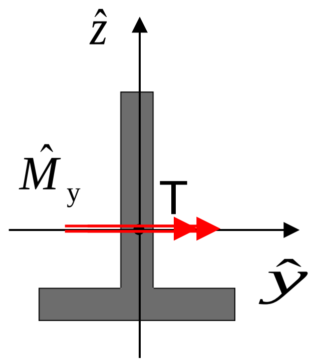
4) obremenitev v obliki sile je usmerjena v smeri “z” koordinatne osi





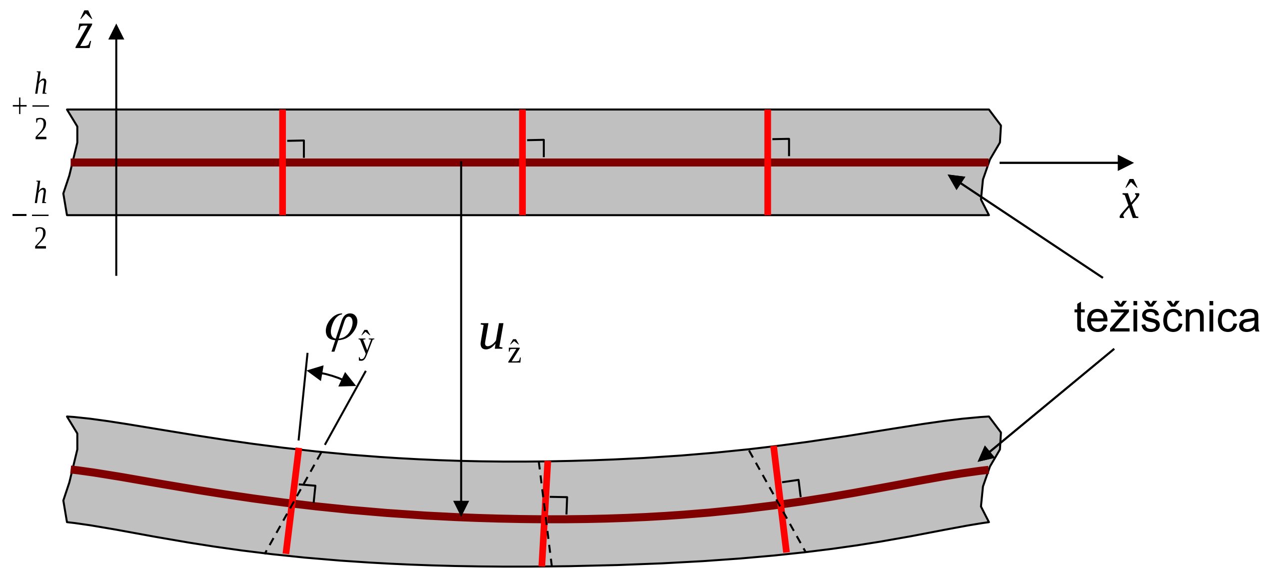
5) obremenitev v obliki momenta je usmerjena okoli “y” koordinatne osi





- upoštevajoč <u>Timoshenk-ovo teorijo nosilcev</u>, lahko pomike u_x v smeri osi nosilca izrazimo z zasukom f_y

$$u_{\hat{x}} = \hat{z}\,\varphi_{\hat{y}}$$



- Timoshenk-ova teorija nosilcev predpostavlja planost prereza v deformiranem stanju, pri čemer pa prerez v splošnem ni več pravokoten na težiščnico

- komponente deformacijskega tenzorja lahko v Kartezijevem koordinatnem sistemu zapišemo v odvisnosti od pomika v smeri "z" koordinatne osi in zasuka okoli "y" koordinatne osi

$$\varepsilon_{\hat{x}\hat{x}} = \frac{\partial u_{\hat{x}}}{\partial \hat{x}} = \hat{z}\,\frac{\partial \varphi_{\hat{y}}}{\partial \hat{x}}$$

$$\varepsilon_{\hat{x}\hat{y}} = 0$$

$$\varepsilon_{\hat{x}\hat{z}} = \frac{1}{2}\left(\frac{\partial u_{\hat{x}}}{\partial \hat{z}} + \frac{\partial u_{\hat{z}}}{\partial \hat{x}}\right) = \frac{1}{2}\left(\varphi_{\hat{y}} + \frac{\partial u_{\hat{z}}}{\partial \hat{x}}\right)$$

$$\varepsilon_{\hat{y}\hat{z}} = 0$$

$$\varepsilon_{\hat{y}\hat{y}} = ?$$

$$\varepsilon_{\hat{z}\hat{z}} = ?$$

$$\begin{Bmatrix} \varepsilon_{\hat{x}\hat{x}} \\ \varepsilon_{\hat{x}\hat{z}} \end{Bmatrix} = \begin{bmatrix} 0 & \hat{z}\,\dfrac{\partial}{\partial \hat{x}} \\ \dfrac{1}{2}\,\dfrac{\partial}{\partial \hat{x}} & \dfrac{1}{2} \end{bmatrix} \begin{Bmatrix} u_{\hat{z}} \\ \varphi_{\hat{y}} \end{Bmatrix} = [L]\{\hat{u}\}$$

- od nič različni komponenti napetostnega tenzorja v Kartezijevem koordinatnem sistemu za primer upogibno obremenjenega nosilca v ravnini (x,z) sta samo σ_{xx} in σ_{xz}

$$\sigma_{ij} = \begin{Bmatrix} \sigma_{\hat{x}\hat{x}} & \sigma_{\hat{x}\hat{y}} = 0 & \sigma_{\hat{x}\hat{z}} \\ \sigma_{\hat{x}\hat{y}} = 0 & \sigma_{\hat{y}\hat{y}} = 0 & \sigma_{\hat{y}\hat{z}} = 0 \\ \sigma_{\hat{x}\hat{z}} & \sigma_{\hat{y}\hat{z}} = 0 & \sigma_{\hat{z}\hat{z}} = 0 \end{Bmatrix}$$

- za homogeni, izotropni, linearno elastični material, lahko iz zveze med napetostmi in deformacijami, ki jo definira Hookov zakon, izračunamo komponenti deformacijskega tenzorja ε_{yy} in ε_{zz}

$$\sigma_{\hat{x}\hat{x}} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{\hat{x}\hat{x}} + \nu\varepsilon_{\hat{y}\hat{y}} + \nu\varepsilon_{\hat{z}\hat{z}}\right]$$

$$\sigma_{\hat{y}\hat{y}} = 0 = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\varepsilon_{\hat{x}\hat{x}} + (1-\nu)\varepsilon_{\hat{y}\hat{y}} + \nu\varepsilon_{\hat{z}\hat{z}}\right]$$

$$\sigma_{\hat{z}\hat{z}} = 0 = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\varepsilon_{\hat{x}\hat{x}} + \nu\varepsilon_{\hat{y}\hat{y}} + (1-\nu)\varepsilon_{\hat{z}\hat{z}}\right]$$

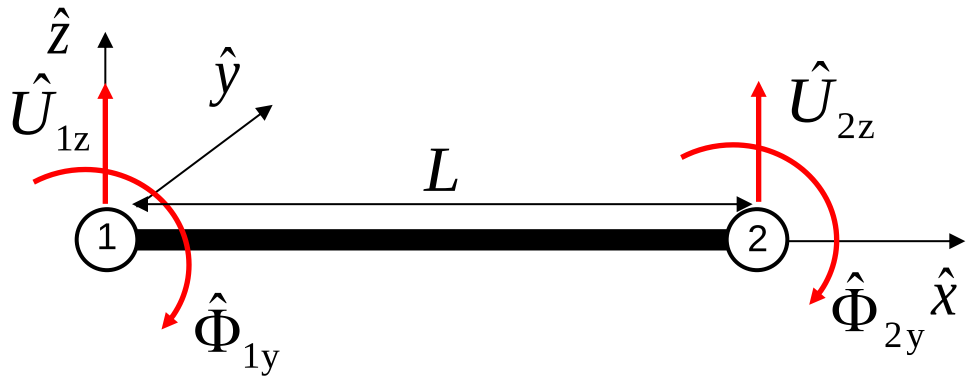
$$\varepsilon_{\hat{y}\hat{y}} = \varepsilon_{\hat{z}\hat{z}}$$

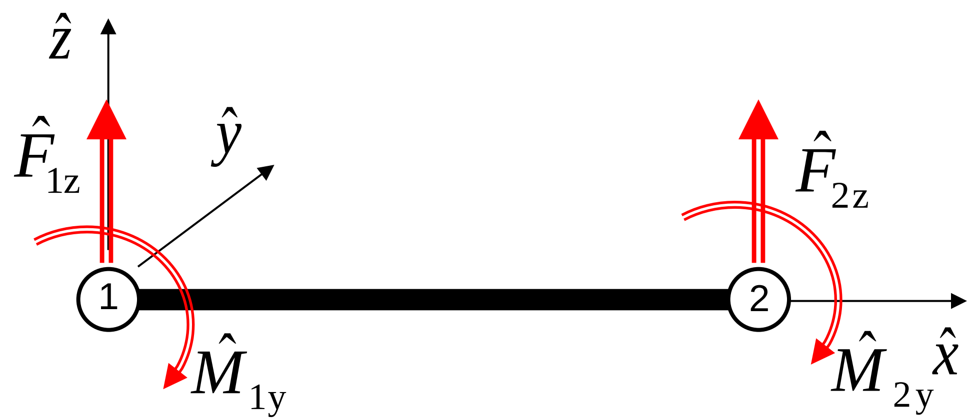
$$\sigma_{\hat{y}\hat{y}} = 0 = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\varepsilon_{\hat{x}\hat{x}} + (1-\nu)\varepsilon_{\hat{y}\hat{y}} + \nu\varepsilon_{\hat{z}\hat{z}}\right] \Rightarrow \varepsilon_{\hat{y}\hat{y}} = \varepsilon_{\hat{z}\hat{z}} = -\nu\varepsilon_{\hat{x}\hat{x}}$$

$$\sigma_{\hat{x}\hat{x}} = E\,\varepsilon_{\hat{x}\hat{x}}$$

- zapis enačb KE v matrični obliki v lokalnem koordinatnem sistemu nosilca, ki se upogiba v ravnini (x,z)

$$\frac{E\,I_{\hat{y}}}{L^3}\begin{bmatrix}\hat{K}_{11} & \hat{K}_{12} & \hat{K}_{13} & \hat{K}_{14}\\ \hat{K}_{21} & \hat{K}_{22} & \hat{K}_{23} & \hat{K}_{24}\\ \hat{K}_{31} & \hat{K}_{32} & \hat{K}_{33} & \hat{K}_{34}\\ \hat{K}_{41} & \hat{K}_{42} & \hat{K}_{43} & \hat{K}_{44}\end{bmatrix}\begin{Bmatrix}\hat{U}_{1z}\\ \hat{\Phi}_{1y}\\ \hat{U}_{2z}\\ \hat{\Phi}_{2y}\end{Bmatrix}=\begin{Bmatrix}\hat{F}_{1z}\\ \hat{M}_{1y}\\ \hat{F}_{1z}\\ \hat{M}_{2y}\end{Bmatrix}$$





- transformacija pomikov in zasukov iz <u>lokalnega</u> v <u>globalni</u> koordinatni sistem obravnavanega problema

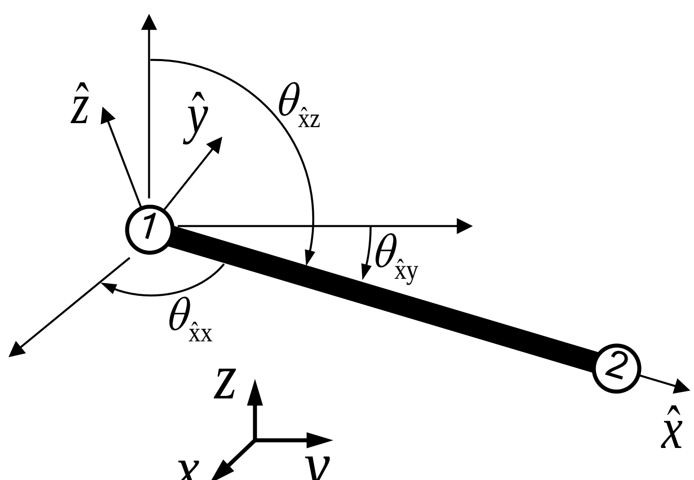
$$\begin{Bmatrix} \hat{U}_{1z} \\ \hat{\Phi}_{1y} \\ \hat{U}_{2z} \\ \hat{\Phi}_{2y} \end{Bmatrix} = \begin{bmatrix} [T] & [0] & [0] & [0] \\ [0] & [T] & [0] & [0] \\ [0] & [0] & [T] & [0] \\ [0] & [0] & [0] & [T] \end{bmatrix} \begin{Bmatrix} U_{1x} \\ U_{1y} \\ U_{1z} \\ \Phi_{1x} \\ \Phi_{1y} \\ \Phi_{1z} \\ U_{2x} \\ U_{2y} \\ U_{2z} \\ \Phi_{2x} \\ \Phi_{2y} \\ \Phi_{2z} \end{Bmatrix}$$

- transformacija sil in momentov iz lokalnega v globalni koordinatni sistem obravnavanega problema

$$\begin{Bmatrix} \hat{F}_{1z} \\ \hat{M}_{1y} \\ \hat{F}_{2z} \\ \hat{M}_{2y} \end{Bmatrix} = \begin{bmatrix} [T] & [0] & [0] & [0] \\ [0] & [T] & [0] & [0] \\ [0] & [0] & [T] & [0] \\ [0] & [0] & [0] & [T] \end{bmatrix} \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{1z} \\ M_{1x} \\ M_{1y} \\ M_{1z} \\ F_{2x} \\ F_{2y} \\ F_{2z} \\ M_{2x} \\ M_{2y} \\ M_{2z} \end{Bmatrix}$$

- transformacija matrika iz lokalnega v globalni koordinatni sistem

$$[T]=\begin{bmatrix} \cos\theta_{\hat{x}x} & \cos\theta_{\hat{x}y} & \cos\theta_{\hat{x}z} \\ \cos\theta_{\hat{y}x} & \cos\theta_{\hat{y}x} & \cos\theta_{\hat{y}x} \\ \cos\theta_{\hat{z}x} & \cos\theta_{\hat{z}x} & \cos\theta_{\hat{z}x} \end{bmatrix}$$



globalni k.s.

- <u>matrični zapis enačbe KE za linearno elastični statično samo upogibno obremenjeni nosilec</u>

- za posamezni KE dobimo toliko enačb, kolikor ima KE <u>prostostnih stopenj</u>
- v vozlišču KE je neznanih <u>šest primarnih veličin</u> – <u>trije pomiki in trije zasuki</u>, tako da ima posamezni KE (6*N_v) <u>prostostnih stopenj</u>

$$[K]_e \{U\}_e = \{F\}_e$$

$$\{U\}_e = \begin{Bmatrix} U_{1x} \\ U_{1y} \\ U_{1z} \\ \Phi_{1x} \\ \Phi_{1y} \\ \Phi_{1z} \\ U_{2x} \\ U_{2y} \\ U_{2z} \\ \Phi_{2x} \\ \Phi_{2y} \\ \Phi_{2z} \end{Bmatrix}_e \qquad \{F\}_e = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{1z} \\ M_{1x} \\ M_{1y} \\ M_{1z} \\ F_{2x} \\ F_{2y} \\ F_{2z} \\ M_{2x} \\ M_{2y} \\ M_{2z} \end{Bmatrix}_e$$

- posamezni element vektorja $\{F\}_e$ predstavlja v vozlišču KE delujočo silo oziroma moment

- v primeru, da je <u>velikost pomika</u> v smeri “z” koordinatne osi v vozlišču KE <u>poznana</u>, velikost sile v tej smeri <u>ni poznana</u>

$$U_{iz} = \checkmark \quad \Rightarrow \quad F_{iz} = ?\,, \quad \mathrm{i} = 1,..,\mathrm{N_v}$$

- v primeru, da je <u>velikost zasuka</u> okoli osi vzporedne “y” koordinatni osi v vozlišču KE <u>poznana</u>, velikost momenta okoli te osi <u>ni poznana</u>

$$\Phi_{iy} = \checkmark \quad \Rightarrow \quad M_{iy} = ?\,, \quad \mathrm{i} = 1,..,\mathrm{N_v}$$

- v primeru, da velikost pomika v vozlišču KE ni poznana, je velikost sile v tej smeri možno izračunati

$$U_{\mathrm{iz}} = ? \quad \Rightarrow \quad F_{\mathrm{iz}} = \checkmark, \quad \mathrm{i} = 1,..,\mathrm{N_v}$$

- v primeru točkovne mehanske obremenitve v obliki sile mrežo KE generiramo tako, da točka, v kateri deluje točkovna obremenitev, sovpada z vozliščem KE

$$F_{\mathrm{Iz}} = F_{\mathrm{Tz}} \;, \quad \mathrm{I} = \{1,..,\mathrm{N_{KE}}\}$$

- v primeru, da <u>velikost zasuka</u> v vozlišču KE <u>ni poznana</u>, je velikost momenta <u>možno izračunati</u>

$$\Phi_{\mathrm{iy}} = ? \quad \Rightarrow \quad M_{\mathrm{iy}} = \checkmark , \quad \mathrm{i} = 1,..,\mathrm{N_v}$$

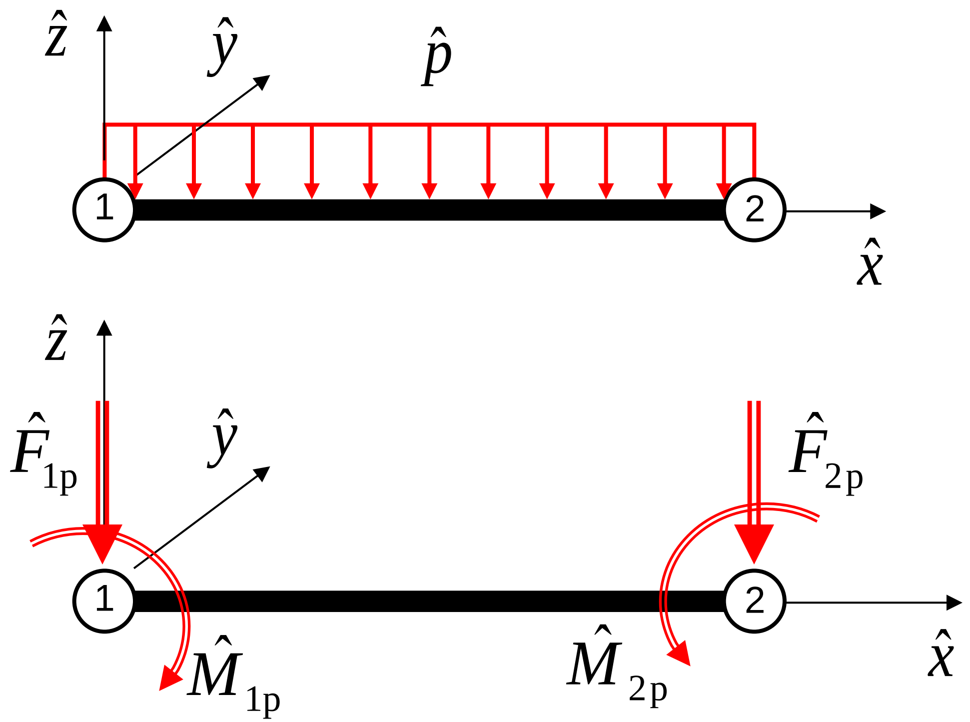
- v primeru <u>točkovne mehanske obremenitve v obliki momenta</u> mrežo KE generiramo tako, da točka, v kateri deluje točkovna momentna obremenitev, sovpada z vozliščem KE

$$M_{\mathrm{Iy}} = M_{\mathrm{Ty}} , \quad \mathrm{I} = \{1,..,\mathrm{N_{KE}}\}$$

- v primeru linijsko porazdeljene mehanske obremenitve, ki deluje v smeri "z" koordinatne osi, izračunamo ekvivalentni vozliščni sili in momenta za posamezni KE

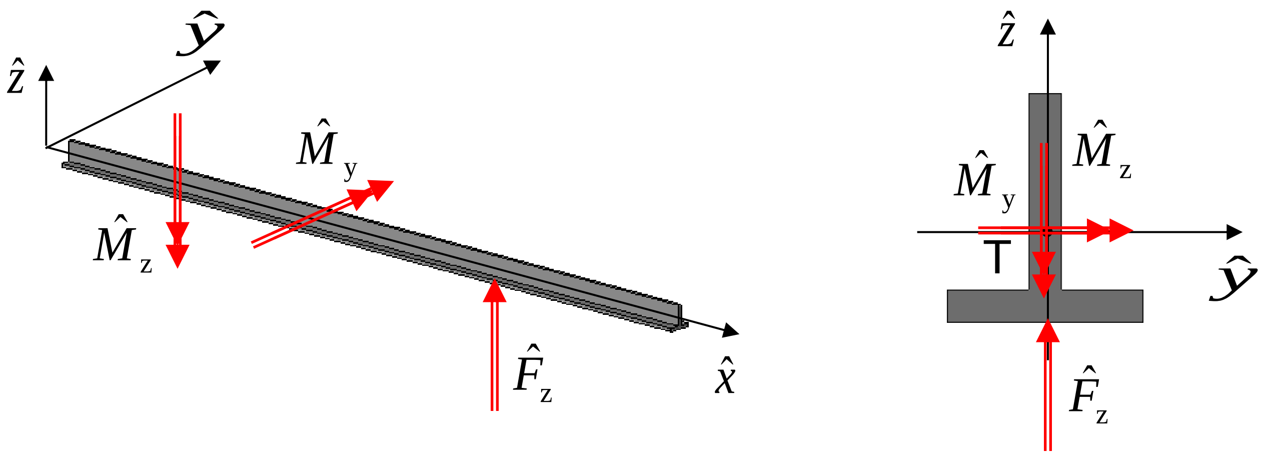
$$\{\hat{F}_{\mathrm{p}}\}_{\mathrm{e}} = \int_0^L \hat{p}\,[N_{\mathrm{U}}]^T \,\mathrm{d}\hat{x}$$

$$\{\hat{M}_{\mathrm{p}}\}_{\mathrm{e}} = \int_0^L \hat{p}\,[N_{\Phi}]^T \,\mathrm{d}\hat{x}$$

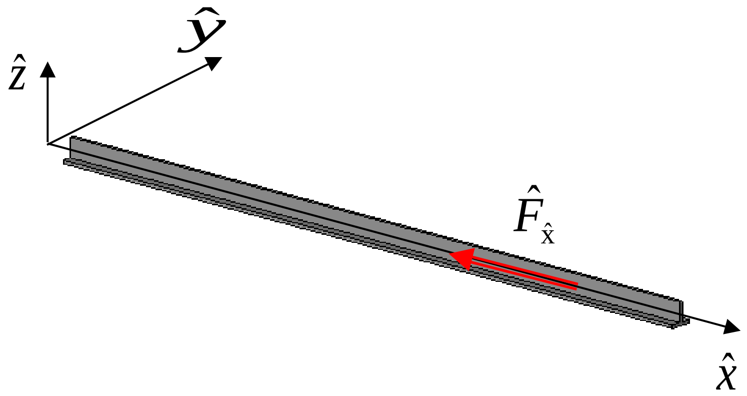


- možne obremenitve nosilca v prostoru:

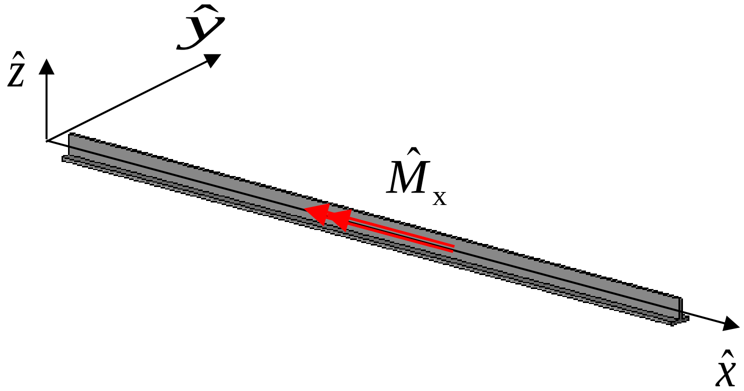
1) <u>upogibna obremenitev</u> - obremenitev pravokotna na os nosilca, pri čemer lahko to obremenitev vedno razdelimo tako, da deluje v dveh, med seboj pravokotnih ravninah

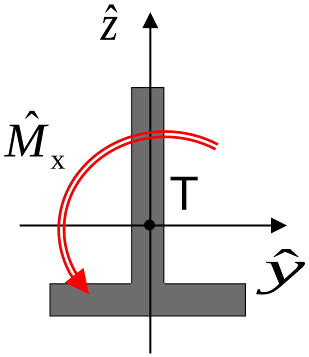


2) osna obremenitev - obremenitev v smeri osi nosilca



3) torzijska - obremenitev okoli osi nosilca





- <u>matrični zapis enačbe KE za linearno elastični statično poljubno obremenjeni nosilec</u>

1) <u>upogibna obremenitev</u> v ravnini (x,y) in (x,z)

$$\frac{E\,I_{\hat{z}}}{L^3}\begin{bmatrix}\hat{K}_{11} & \hat{K}_{12} & \hat{K}_{13} & \hat{K}_{14}\\ \hat{K}_{21} & \hat{K}_{22} & \hat{K}_{23} & \hat{K}_{24}\\ \hat{K}_{31} & \hat{K}_{32} & \hat{K}_{33} & \hat{K}_{34}\\ \hat{K}_{41} & \hat{K}_{42} & \hat{K}_{43} & \hat{K}_{44}\end{bmatrix}\begin{Bmatrix}\hat{U}_{1y}\\ \hat{\Phi}_{1z}\\ \hat{U}_{2y}\\ \hat{\Phi}_{2z}\end{Bmatrix}=\begin{Bmatrix}\hat{F}_{1y}\\ \hat{M}_{1z}\\ \hat{F}_{1y}\\ \hat{M}_{2z}\end{Bmatrix}$$

$$\frac{E\,I_{\hat{y}}}{L^3}\begin{bmatrix}\hat{K}_{11} & \hat{K}_{12} & \hat{K}_{13} & \hat{K}_{14}\\ \hat{K}_{21} & \hat{K}_{22} & \hat{K}_{23} & \hat{K}_{24}\\ \hat{K}_{31} & \hat{K}_{32} & \hat{K}_{33} & \hat{K}_{34}\\ \hat{K}_{41} & \hat{K}_{42} & \hat{K}_{43} & \hat{K}_{44}\end{bmatrix}\begin{Bmatrix}\hat{U}_{1z}\\ \hat{\Phi}_{1y}\\ \hat{U}_{2z}\\ \hat{\Phi}_{2y}\end{Bmatrix}=\begin{Bmatrix}\hat{F}_{1z}\\ \hat{M}_{1y}\\ \hat{F}_{1z}\\ \hat{M}_{2y}\end{Bmatrix}$$

2) osna obremenitev

$$\frac{E\,A}{L}\begin{bmatrix}+1 & -1\\ -1 & +1\end{bmatrix}\begin{Bmatrix}\hat{U}_{1x}\\ \hat{U}_{2x}\end{Bmatrix}=\begin{Bmatrix}\hat{F}_{1x}\\ \hat{F}_{2x}\end{Bmatrix}$$

3) torzijska obremenitev

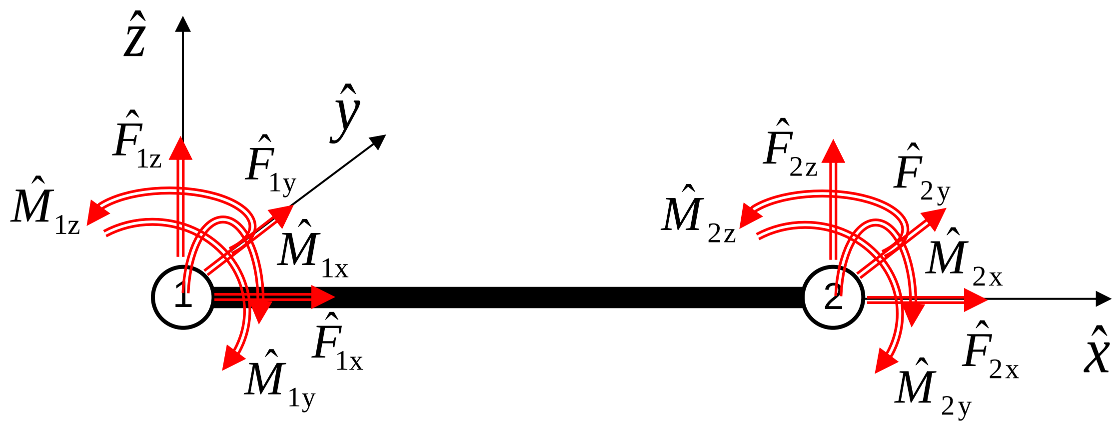
$$\frac{G\,J_{\hat{x}}}{L}\begin{bmatrix}+1 & -1\\ -1 & +1\end{bmatrix}\begin{Bmatrix}\hat{\Phi}_{1x}\\ \hat{\Phi}_{2x}\end{Bmatrix}=\begin{Bmatrix}\hat{M}_{1x}\\ \hat{M}_{2x}\end{Bmatrix}$$

- zapis sistema enačb za KE v lokalnem koordinatnem sistemu

$$
\begin{bmatrix}
\left[\hat{K}_{\text{osna}}\right] & [0] & [0] & [0] \\
[0] & \left[\hat{K}_{\text{upogib (x, y)}}\right] & [0] & [0] \\
[0] & [0] & \left[\hat{K}_{\text{upogib (x, z)}}\right] & [0] \\
[0] & [0] & [0] & \left[\hat{K}_{\text{torzija}}\right]
\end{bmatrix}
\begin{Bmatrix}
\hat{U}_{1x} \\ \hat{U}_{2x} \\ \hat{U}_{1y} \\ \hat{\Phi}_{1z} \\ \hat{U}_{2y} \\ \hat{\Phi}_{2z} \\ \hat{U}_{1z} \\ \hat{\Phi}_{1y} \\ U_{2z} \\ \Phi_{2y} \\ \hat{\Phi}_{1x} \\ \hat{\Phi}_{2x}
\end{Bmatrix}_e
=
\begin{Bmatrix}
\hat{F}_{1x} \\ \hat{F}_{2x} \\ \hat{F}_{1y} \\ \hat{M}_{1z} \\ \hat{F}_{2y} \\ \hat{M}_{2z} \\ \hat{F}_{1z} \\ \hat{M}_{1y} \\ \hat{F}_{2z} \\ \hat{M}_{2y} \\ \hat{M}_{1x} \\ \hat{M}_{2x}
\end{Bmatrix}_e
$$

- primarne in sekundarne veličine v vozliščih KE v lokalnem koordinatnem sistemu

primarne veličine



sekundarne veličine

- sistem enačb za obravnavani KE za reševanje linearno elastičnih statično obremenjenih problemov

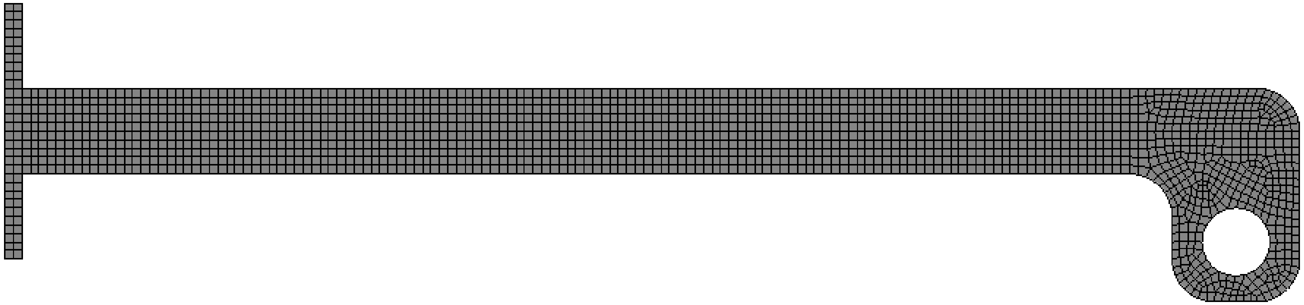
- za posamezni KE dobimo toliko enačb, kolikor ima KE prostostnih stopenj

- pri preslikavi iz lokalnega koordinatnega sistema KE v globalni koordinatni sistem dobimo v posameznem vozlišču KE šest primarnih veličin – tri pomike v smereh globalnih koordinatnih osi in tri zasuke okoli globalnih koordinatnih osi

- v vozlišču KE je tako šest primarnih veličin, tako da ima posamezni KE ($6*N_v$) prostostnih stopenj

- primer reševanja upogibno obremenjenega nosilca z MKE

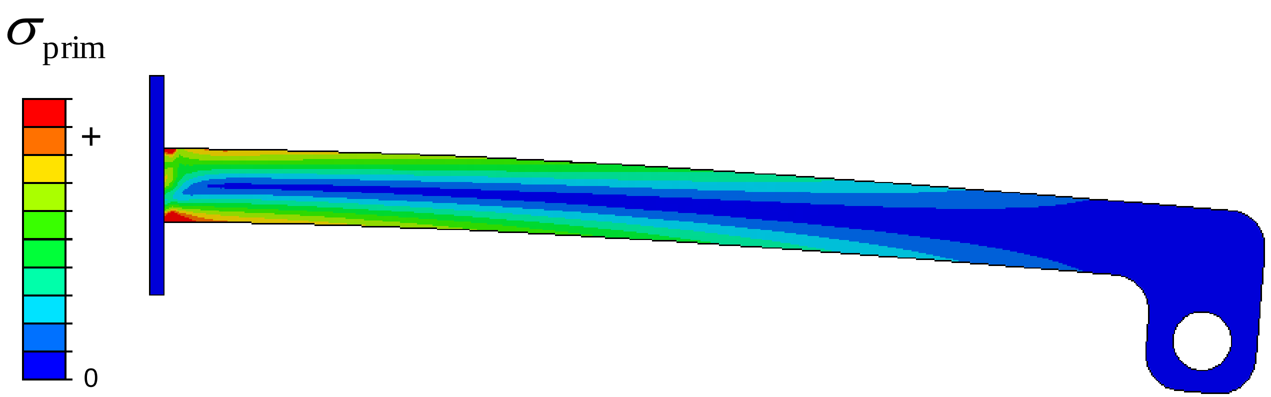
3D KE:
16400 KE (6 pl., 8 vozl.)
12300 vozlišč
36900 enačb



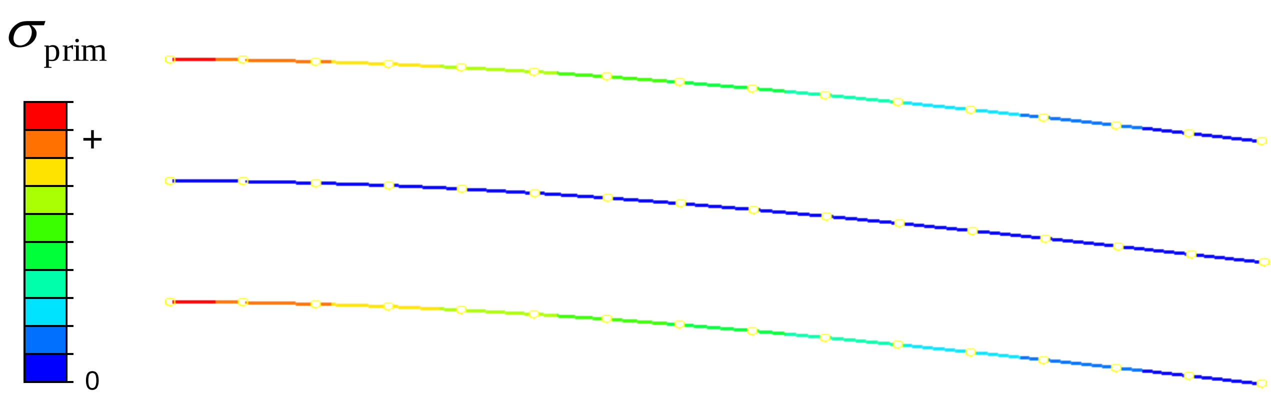
1D KE:
15 KE (2 vozl.)
16 vozlišč
96 enačb

- primerjava Mises-ove primerjalne napetosti: 3D KE ⟷ 1D KE

3D KE

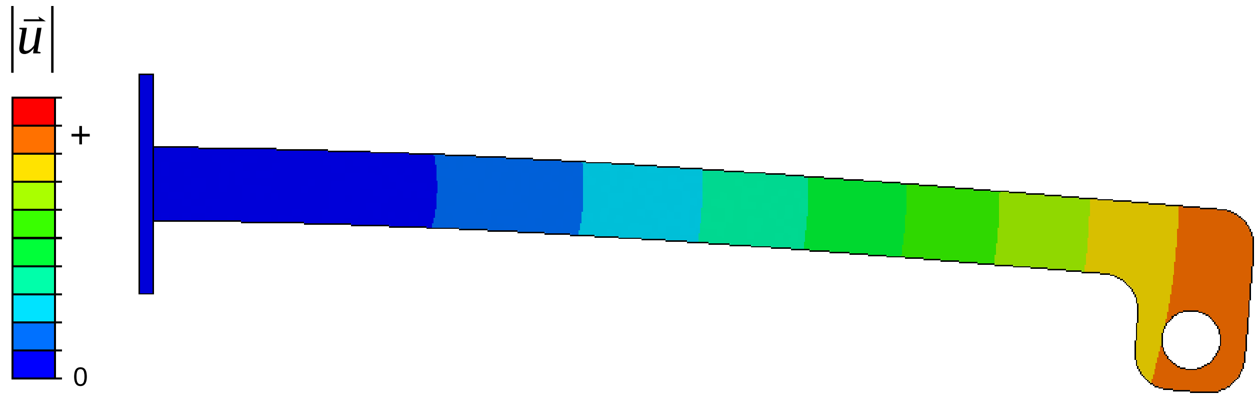


1D KE



- primerjava pomikov : <u>3D KE</u> ⟷ <u>1D KE</u>

3D KE



1D KE